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Solution of Temperature Fluid Particle in Incompressible Dusty Fluid with The Effect of Week Induced Magnetic Field

Abstract— It is possible for dust particles to naturally exist in fluids. In the study of fluid mechanics, these problems related to flow characteristics of temperature. How the magnetic field of suspended particulate matter affects the temperature axially symmetrical jet mixing of incompressible dusty fluid. we assume that the velocity and temperature in the jet deviate from the surrounding stream. To linearize the equation that was solved using Laplace Transformation, a perturbation method was used. The solution of temperature of the particle phase which is depends on fluid phase temperature with in the weak induced Magnetic field.

Keyword: Induced Magnetic field, Differential equations, Dusty fluid flow, incompressible fluid.

I. INTRODUCTION

We discussed about the interaction of an induced magnetic field with a temperature fluid particle in an axisymmetric jet mixing an incompressible fluid in cylindrical polar coordinates. The governing differential equations have been linearized using the perturbation method under the assumption that the velocity and temperature in the jet are only slightly different from those of the surrounding stream. The Laplace transformation technique has been used to solve the resulting linearized equations. In order to discuss the profiles of perturbation particle temperature, numerical calculations have been undertaken. Finite volume fraction analysis reveals a considerable reduction in the size of the perturbation particle temperature.

A. Mathematical Formulation

The equation governing the study two-phase boundary layer flow in axi-symmetric case can be written in cylindrical polar coordinates as

Heat Equation in Fluid phase

$$(1-\phi)\rho C_p \left(u \frac{\partial T}{\partial z} + v \frac{\partial T}{\partial r} \right) = K \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) + \rho_p C_s \frac{(T_p - T)}{\tau_t} \quad (1)$$

Heat Equation in Particle phase

$$\rho_p C_p \left(u_p \frac{\partial T_p}{\partial z} + v_p \frac{\partial T_p}{\partial r} \right) = -\rho_p C_s \frac{(T_p - T)}{\tau_T} \quad (2)$$

To study the boundary layer flow, we introduce the dimensionless variables are

$$\bar{z} = \frac{z}{\lambda}, \quad \bar{r} = \frac{r}{(\tau_m \nu)^{\frac{1}{2}}}, \quad \bar{u} = \frac{u}{U}, \quad \bar{v} = v \left(\frac{\tau_m}{\nu} \right)^{\frac{1}{2}}, \quad \bar{u}_p = \frac{u_p}{U}, \quad \bar{v}_p = v_p \left(\frac{\tau_m}{\nu} \right)^{\frac{1}{2}}, \quad \alpha = \frac{\rho_{p0}}{\rho} = const$$

$$\bar{\rho}_p = \frac{\rho_p}{\rho_{p0}}, \quad \bar{T} = \frac{T}{T_0}, \quad \bar{T}_p = \frac{T_p}{T_0},$$

$$\lambda = \tau_m U, \quad \tau_m = \frac{2}{3} \frac{C_p}{C_s} \frac{1}{p_r} \tau_T, \quad p_r = \frac{\mu C_p}{K}$$

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We may now assume that the pressure in the mixing region is roughly constant while taking the flow from the orifice during complete expansion into account. As a result, the pressure at the exit is the same as the stream's surrounding pressure. As a result, the jet's velocity and temperature are little different from those of the stream around it. Both the thermal conductivity K and the coefficient of viscosity are taken to be constants. Then,

$$\begin{aligned} u &= u_0 + u_1, \\ v &= v_1, \\ u_p &= u_{p_0} + u_{p_1}, \\ v_p &= v_{p_1}, T = T_0 + T_1, T_p = T_{p_0} + T_{p_1}, \rho_p = \rho_{p_1} \end{aligned}$$

where the subscripts 1 denotes the perturbed values which are much smaller than the basic values with subscripts '0' of the surrounding stream, i.e.

$$u_0 \gg u_1, u_{p_0} \gg u_{p_1}, T_0 \gg T_1, T_{p_0} \gg T_{p_1}.$$

Using the dimensionless variable and the perturbation method the nonlinear equations (1) and (2) becomes after dropping the bar can be written as

$$\begin{aligned} (1-\phi) u_0 \frac{\partial T_1}{\partial z} &= \frac{1}{p_r} \left(\frac{\partial^2 T_1}{\partial r^2} + \frac{1}{r} \frac{\partial T_1}{\partial r} \right) \\ &+ \frac{2\alpha}{3p_r} \rho_{p_1} (T_{p_0} - T_0) \end{aligned} \quad (3)$$

$$u_{p_0} \frac{\partial T_{p_1}}{\partial z} = \frac{2}{3} \frac{1}{p_r} \left[(T_0 - T_{p_0}) + (T_1 - T_{p_1}) \right] \quad (4)$$

The boundary conditions for u_1, v_1, u_{p_1} and v_{p_1} are

$$u_{p_1}(0, r) = \begin{cases} u_{p_{10}}, & r \leq 1 \\ 0, & r > 1 \end{cases} \quad (5)$$

Similarly the boundary conditions for T_1, T_{p_1} are

$$T_1(0, r) = \begin{cases} T_{10}, & r \leq 1 \\ 0, & r > 1 \end{cases} \quad (6)$$

$$\frac{\partial T_1}{\partial r}(z, 0) = 0, T_1(z, \infty) = 0 \quad (7)$$

$$T_{p_1}(0, r) = \begin{cases} T_{p_{10}}, & r \leq 1 \\ 0, & r > 1 \end{cases} \quad (8)$$

The boundary conditions for particle density ρ_{p_1} are

$$\rho_{p_1}(0, r) = \begin{cases} \rho_{p_{10}}, & r \leq 1 \\ 0, & r > 1 \end{cases} \quad (9)$$

Method Of Solution

The governing linearized equation (4) have been solved by using Laplace transform technique and using the relevant conditions from (5) to (9) we get

$$u_{p_0} \frac{\partial T_{p_1}}{\partial z} = \frac{2}{3p_r} \left[(T_0 - T_{p_0}) + (T_1 - T_{p_1}) \right]$$

Taking Laplace Transform on both sides

$$\begin{aligned} \Rightarrow u_{p_0} L \left(\frac{\partial T_{p_1}}{\partial z} \right) &= \frac{2}{3p_r} \left[L(T_0 - T_{p_0}) + L(T_1 - T_{p_1}) \right] \\ \Rightarrow \frac{\partial T_{p_1}^*}{\partial z} &= \frac{2}{3p_r u_{p_0}} K + \frac{2}{3p_r u_{p_0}} T_1^* - \frac{2}{3p_r u_{p_0}} T_{p_1}^* \end{aligned}$$

$$\text{where } K = \frac{T_0 - T_{p_0}}{p}$$

$$\Rightarrow \frac{\partial T_{p_1}^*}{\partial z} + \frac{2}{3p_r u_{p_0}} T_{p_1}^* = \frac{2}{3p_r u_{p_0}} (K + T_1^*)$$

$$\Rightarrow \frac{\partial T_{p_1}^*}{\partial z} + C T_{p_1}^* = C (K + T_1^*)$$

$$\text{Where } C = \frac{2}{3p_r u_{p_0}}$$

Which is linear first order differential equation.

To obtain the solution ,

$$I.F = e^{\int C dz}$$

The required Solution is

$$\begin{aligned} T_{p_1}^* e^{\int C dz} &= \int C (K + T_1^*) e^{\int C dz} dz \\ &= e^{-\int C dz} \int C (K + T_1^*) e^{\int C dz} dz \end{aligned}$$

$$= e^{-\int C dz} \int C (K + T_1^*) e^{cz} dz$$

$$T_{p_1}^* = e^{-cz} \left[C (K + T_1^*) \frac{e^{cz}}{c} - \int \frac{\partial T_1^*}{\partial z} e^{cz} dz \right]$$

We have

$$T_1^* = \left(T_{10} - \frac{2\alpha E \rho_{p10}}{3Ap^2} \right) \frac{J_1(p)}{p} e^{-\frac{Ap^2 z}{Ap^2}} + \frac{2\alpha E \rho_{p10}}{3Ap^2} \frac{J_1(p)}{p}$$

Now

$$\int e^{cz} \frac{\partial T_1^*}{\partial z} dz = \int - \left(\frac{AT_{10}p^2}{p_r} - \frac{2\alpha F \rho_{p10}}{3p_r} \right) \frac{J_1(p)}{p} e^{-\frac{Ap^2 z}{p_r}} e^{cz} dz$$

$$= - \int \left(\frac{AT_{10}p^2}{p_r} - \frac{2\alpha F \rho_{p10}}{3p_r} \right) \frac{J_1(p)}{p} e^{\left(c - \frac{Ap^2}{p_r} \right) z} dz$$

$$= \left(\frac{AT_{10}p^2}{p_r} - \frac{2\alpha F \rho_{p10}}{3p_r} \right) \frac{J_1(p)}{p} \frac{e^{\left(c - \frac{Ap^2}{p_r} \right) z}}{c - \frac{Ap^2}{p_r}} + D$$

$$T_{p_1}^* (z, p) = T_1^* (z, p) + \left(\frac{AT_{10}p^2}{p_r} - \frac{2\alpha F \rho_{p10}}{3p_r} \right) \frac{J_1(p)}{p} \frac{e^{\left(c - \frac{Ap^2}{p_r} \right) z}}{c - \frac{Ap^2}{p_r}} + D e^{-cz}$$

When $z \rightarrow 0$ we have

$$T_{p_1}^* (0, p) = T_1^* (0, p) + \left(\frac{AT_{10}p^2}{p_r} - \frac{2\alpha F \rho_{p10}}{3p_r} \right) \frac{J_1(p)}{p} \frac{1}{c - \frac{Ap^2}{p_r}} + D$$

$$D = \frac{T_{p10} J_1(p)}{p} - \frac{T_{10} J_1(p)}{p} - \left(\frac{AT_{10}p^2}{p_r} - \frac{2\alpha F \rho_{p10}}{3p_r} \right) \frac{J_1(p)}{p} \frac{1}{c - \frac{Ap^2}{p_r}}$$

Using the above equation, we have

$$T_{p_1}^* (z, p) = \left(T_{10} - \frac{2\alpha F \rho_{p10}}{3Ap^2} \right) \frac{J_1(p)}{p} e^{-\frac{Ap^2 z}{p_r}} + \frac{2\alpha F \rho_{p10}}{3Ap^2} \frac{J_1(p)}{p}$$

$$+ \left(\frac{AT_{10}p^2}{p_r} - \frac{2\alpha F \rho_{p10}}{3p_r} \right) \frac{J_1(p)}{p} \frac{e^{-\frac{Ap^2 z}{p_r}}}{c - \frac{Ap^2}{p_r}} - (T_{10} - T_{p10}) \frac{J_1(p)}{p} e^{-cz} - \left(\frac{AT_{10}p^2}{p_r} - \frac{2\alpha F \rho_{p10}}{3p_r} \right) \frac{J_1(p)}{p} \frac{e^{-cz}}{c - \frac{Ap^2}{p_r}}$$

$$= \frac{-J_1(p) e^{-cz}}{p \left(1 - \frac{Ap^2}{p_r} \right)}$$

$$\left[\frac{AT_{10}p^2}{p_r} - \frac{2\alpha F \rho_{p10}}{3p_r} + T_{10} \left(c - \frac{Ap^2}{p_r} \right) - T_{p10} \left(c - \frac{Ap^2}{p_r} \right) \right] + \frac{2\alpha F \rho_{p10}}{3pr} \frac{J_1(p)}{p} + \frac{J_1(p) e^{-\frac{Ap^2 z}{p_r}}}{p \left(c - \frac{Ap^2}{p_r} \right)}$$

$$\left[\frac{AT_{10}p^2}{p_r} - \frac{2\alpha F \rho_{p10}}{3p_r} + \left(c - \frac{Ap^2}{p_r} \right) \left(T_{10} - \frac{2\alpha F \rho_{p10}}{3p_r} \right) \right] \left[\frac{AT_{10}p^2}{p_r} - \frac{2\alpha F \rho_{p10}}{3p_r} + T_{10} \left(c - \frac{Ap^2}{p_r} \right) - T_{p10} \left(c - \frac{Ap^2}{p_r} \right) \right]$$

$$T_{p_1}^* = \left[\left(T_{p10} + \frac{\frac{2\alpha F \rho_{p10}}{3Cp_r} - T_{10}}{1 - \frac{Ap^2}{Cp_r}} \right) e^{-cz} + \frac{2\alpha F \rho_{p10}}{3Ap^2} + \left(\frac{T_{10} - \frac{2\alpha F \rho_{p10}}{3Ap^2}}{1 - \frac{Ap^2}{Cp_r}} \right) e^{-\frac{Ap^2 z}{p_r}} \right] \frac{J_1(p)}{p}$$

Where $u_1^* = \int_0^\infty r u_1 J_0(pr) dr$ etc.

$$A = \frac{1}{(1-\phi)u_0}, E = \frac{u_{p_0} - u_0}{u_0}, C = \frac{2}{3p_r u_{p_0}}, F = \frac{T_{p_0} - T_0}{(1-\phi)u_0}$$

And

$$\rho_{p_1}^* = \rho_{p_0} \frac{J_1(p)}{p}$$

$$T_{p1}^* = \left[\left(T_{p10} + \frac{2\alpha F \rho_{p10} - T_{10}}{3Cp_r} \right) e^{-cz} + \frac{2\alpha F \rho_{p10}}{3Ap^2} + \left(\frac{T_{10} - \frac{2\alpha F \rho_{p10}}{3Ap^2}}{1 - \frac{Ap^2}{Cp_r}} \right) e^{-\frac{Ap^2 z}{Pr}} \right] \frac{J_1(p)}{p} \quad (10)$$

Where J_0 and J_1 are the Bessel function of zero and first order respectively.

DISCUSSION OF RESULT AND CONCLUSION

Numerical computation has been made by taking $Pr = 0.72$, $u_{10} = up_{10} = T_{10} = Tp_{10} = \rho_{p10} = 0.1$, $\phi = 0.01$. The velocity of temperature at the exit are taken nearly equal to unity. Equation (10) is the solution of temperature of the particle phase which is depends on fluid phase temperature with in the weak induced Magnetic field.

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