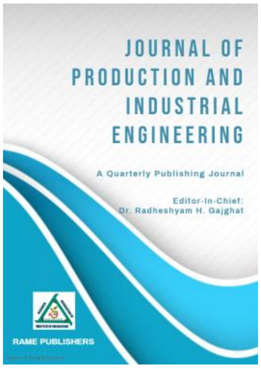
Journal of Production and Industrial Engineering
Volume 5, Issue 2, December 2024, pp 48 - 58
https://doi.org/10.26706/jpie.5.2.20240902

Determining the Weights of Integer Linear Programming Model using the Analytic Hierarchical Process (AHP)

Ansseif A.latif Ansseif [1,*], Aseel Nasser Sabti[2] , Zaid A.Mohammed[3]

[1,2,3] College of Administration and Economics, Al-Iraqia University, Baghdad, Iraq.

* Correspondence: nsseif.latif@aliraqia.edu.iq



Article/ Review – Peer Reviewed
Received: 17 Sept 2024
Accepted: 20 Nov 2024
Published: 30 Dec 2024

Cite this article: Ansseif A.latif Ansseif, Aseel Nasser Sabti, Zaid A.Mohammed, "Determining the Weights of Integer Linear Programming Model using the Analytic Hierarchical Process (AHP)", *Journal of Production and Industrial Engineering*, RAME Publishers, vol. 5, issue 2, pp. 48-58, 2024.
https://doi.org/10.26706/ijceae.5.2.20240902

Abstract: This research presents a well-studied quantitative method based on Integer Linear Programming (ILP) in order to determine the effect of the available alternatives in the company's products through the relative importance weights of the sample variables and based on the Analytic Hierarchical Process (AHP) without resorting to speculative methods in the process of calculating and weighting the weights of the decision variables. investigated sample. Therefore, we will discuss the application of the (AHP) method through a questionnaire prepared for the purpose of determining the Pairwise Comparisons Matrices and the relative importance of a set of criteria and alternatives in the research sample, and thus using it to feed the mathematical model (ILP) to know the effect of these alternatives on the company's products as well maximize the weighting of the available alternatives to take the optimal decision.

Keywords: Analytic Hierarchical Process (AHP); Integer Linear Programming (ILP); Pairwise Comparisons Matrices; Eigenvector.

1. Introduction

In view of the dynamism and rapid development that surrounds industrial or service institutions; it has become difficult for that administration to solve the problems it faces by relying on the method of intuition and guesswork. Therefore, these institutions realized the necessity of using models and quantitative methods in the decision-making process, especially that the practical reality required solving problems and achieving several goals at the same time.[1]

Therefore, decision makers resorted to using and developing special quantitative methods that address multifactorial decision problems and criteria, meaning that the decision maker has to choose between several alternatives.[2]. There are many applications of Integer Linear Programming (ILP) and analytic hierarchical process in the available literature for example [3] They submitted an approach based on linear programming (LP) that estimates the weights for a pairwise comparison matrix generated within the framework of the analytic hierarchy process and LP models were formulated for many matrices of pair comparisons and the results of LP models were compared with two widely used methods EM and LLS. Through LP models, the same weights were obtained from the two methods.[4] improved a multiple objective mixed integer stochastic programming (SMIP) model for the vendor selection problem (VSP) with stochastic demand under multi-products purchases. [5] He proposed a model of selecting the right vendor from a group of vendors using the Integer Linear Programming (ILP) model for multiple products and this model was resolved as a decision support system according to the visions and plans of a company. [6]

In this paper, the researchers proposed a fuzzy model based on integer linear programming and also based on the development of the fuzzy analytic hierarchical process (FAHP) by using an integrated multiple criteria decision making (MCDM) method for the contemporary transshipment problem.[7] The researchers used Linear Programming (LP) models to estimate the weights of the matrix of pairwise comparisons in the framework of the analytic hierarchical process (AHP), where the (priorities) that were obtained represented the coefficients of the objective function of the linear programming model in improving the human resource problem. [8] The main objective of this article is to propose the design of the supply chain to achieve the optimum utilization of the resources of an organization and to structure a system for its drivers to achieve responsiveness and efficiency, as well as to include conflicting performance measures that have been addressed through a combination of the analytic hierarchical process (AHP) and Goal Programming (GP) to determine weights. [9] In this paper, the researchers proposed a two-stage model for the design of a bio-network and supply chain for algae-based products. In the first phase, the sites for algae cultivation are identified through the analytic hierarchical process (AHP), and in the second phase, a mixed integer linear programming (MILP) model was developed, the study was conducted in Iran.

2. Multi-Criteria Decision Making (MCDM)

Multi-criteria decision-making is one of the most prominent methods known among the methods of decision-making, and it is a branch of operations research sciences, where it is concerned with solving decision problems that include several multi-criteria with the possibility of taking into account that these goals may be contradictory sometimes resulting from multiple interests And different points of view and problems of a complex nature as a result of the lack of data related to the problem, as well as the difficulty of determining the importance of one criterion without the other. [10] In the past and before the emergence of methods for analyzing multi-criteria decisions, decision-making problems were mostly based on one criterion or objective function, either maximizing profits or reducing costs, either economic, social or environmental problems, for example, do not depend on one goal or criterion only, but It goes beyond that. For example, the recruitment process in an institution will be selected based on the certificate, years of experience, foreign language proficiency, age ... and others. In hospitals, the multi-criteria decision will include several elements, including: reducing costs, improving quality, health services ..., so it was It is more appropriate to resort to methods that include several criteria and several restrictions, which are multi-criteria decision-making methods.[11]

It can be noted that these methods include quantitative and qualitative criteria at the same time, and often do not have the same importance in the decision-making process.[12]

3. The Analytic hierarchical process (AHP)

One of the success of organizations is the use of appropriate methods for decision-making, especially if the decision falls within the management of the organization and therefore it will be the smallest decision that can cause large and influential results, whether positive or negative, and the Analytic Hierarchical Process (AHP) is one of the most multi-criteria decision-making methods Its purpose is to help people organize their thoughts and judgments to make more effective decisions and is based in its structure on observations of how influences are transmitted and calculated, and is derived from the observations of psychologists to understand people's behavior.[13]

Saaty (1980) (AHP) defined “as an integrated framework that combines objective and non-objective criteria with pairwise comparisons based on a relative scale”. [14]

4. Algorithm of (AHP)

The Analytic Hierarchical Process consists of a number of logical steps that include defining the problem and structuring it hierarchically, building a binary comparison matrix, deriving the standard matrix to determine priorities, and finally measuring consistency and stability of judgments. [15]

Step 1: Define and structure the problem hierarchically

The success of building the hierarchical structure of a problem, is the accurate and broad search for all dimensions of the problem under study from a group of specialists and experience to identify the relevant elements and alternatives and arrange them in a hierarchical manner [16] Any graphic representation of the levels and stages of the problem, starting from the first level, which represents the general goal, and the criteria represented in the second level. As for the decision alternatives at the third level, and Figure (1) shows the general hierarchical structure of a problem: [13]

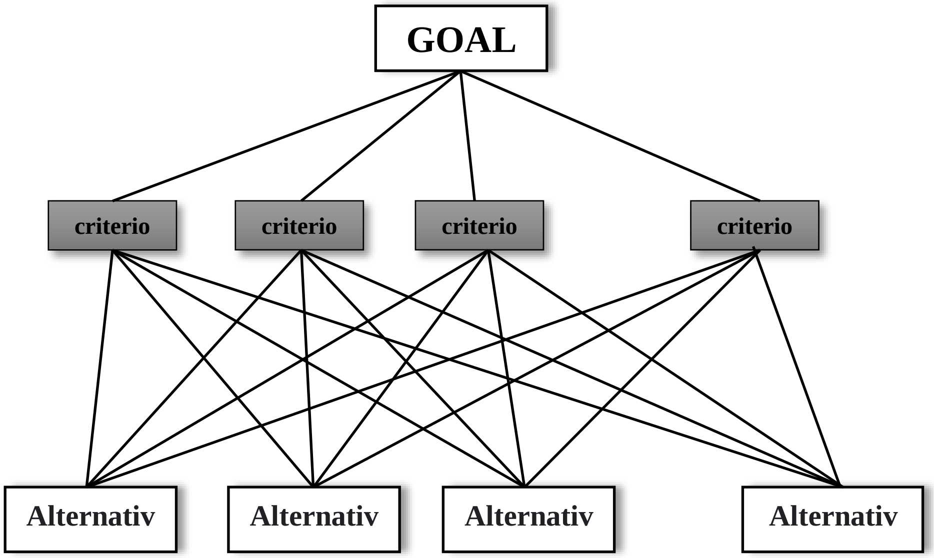


Figure 1. hierarchical levels

The hierarchical structure in Figure (1) reflects the relationship between different factors in the nature of the problem being studied. the effectiveness and influence of the factors of the lower hierarchical level on the factors of the higher level is determined by calculating the percentage of each factor in the hierarchical structure and the evaluation of alternatives depends on the higher levels of the pyramid. and to reduce the calculations as well as the stability of the results. The number of alternatives in each group should not exceed seven [17] [18]

Step 2: Pairwise Comparisons Matrix

The pairwise comparison process is done by making comparisons once between each pair of alternatives with each of the criteria, and again between each pair of criteria with the general objective of the problem in light of the use of the nine-degree preference scale as shown in Table (1), in order to determine The relative importance of each pairwise comparison resulting from measuring the effect (as a numerical relationship) between the alternatives and the criteria in achieving the desired goal. [19]

Table 1. measures (saaty) for the levels of relative importance in the pairwise comparisons

Priority Level	Definition	Illustration
1	Equal importance	Both activities contribute equally to achieving the goals
2	weak significance	
3	Medium importance	A slight preference for one activity over the other
4	More than average importance	
5	strong importance	Strong preference for one activity over another
6	More important than strong	
7	very strong importance	Preferring one activity over another very much
8	The importance of more than very strong	
9	very important	Preferring one activity over another to a maximum degree

When comparing two criteria or alternatives and giving for example the degree of importance (5) in the pairwise comparison matrix while the inverse value of the same two criteria or alternatives is (1/5), but in the case of comparing the alternative or criterion with itself it will be equal to (1), Therefore, the diameter of the matrix will be equal to (1), as shown in the matrix below [20]

Assuming that:
W_i = Criteria weight
n = No. of Criteria
a_{ij} = The ratio of the importance of element (i) to element (j)

The pairwise comparison matrix (A), which is a symmetric matrix, can be expressed as follows: [21]

$$A = \begin{bmatrix} W_1/W_1 & W_1/W_2 & \cdots & W_1/W_n \\ W_2/W_1 & W_2/W_2 & \cdots & W_2/W_n \\ \vdots & \vdots & \ddots & \vdots \\ W_n/W_1 & W_n/W_2 & \cdots & W_n/W_n \end{bmatrix} \quad \text{….(1)}$$

Now
$$a_{ij} = \frac{w_i}{w_j} \quad \text{… (2)}$$

$$= \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \quad \text{… (3)}$$

And
$$a_{ij} = \frac{1}{a_{ji}} \quad \text{… (4)}$$

$$= \begin{bmatrix} 1 & a_{12} & \cdots & a_{1n} \\ \frac{1}{a_{12}} & 1 & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{a_{1n}} & \frac{1}{a_{2n}} & \cdots & 1 \end{bmatrix} \quad \text{… (5)}$$

Step 3: Normalizing Matrix the pair-wise comparison

The standard matrix is derived based on the pairwise comparison matrix by following the following operations: [20]

1- Add the values of each column in the pairwise comparison matrix
2- Divide each element in the pairwise comparison matrix by the sum of the respective column
3- The sum of all the columns in the standard matrix must be equal (1)
4- Finding the average of the rows by collecting the values of the elements of each row in the benchmarking matrix and dividing them by the number of those elements to represent W_n (the relative weights of each row) the priorities that the decision maker wants to determine.

The above steps can be represented by the matrix N and my agencies:

$$N = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \\ \sum \boldsymbol{a_{n1}} & \sum \boldsymbol{a_{n2}} & \cdots & \sum \boldsymbol{a_{nn}} \end{bmatrix} \quad \text{…. (6)}$$

$$= \begin{bmatrix} \frac{a_{11}}{\sum a_{n1}} & \frac{a_{12}}{\sum a_{n2}} & \cdots & \frac{a_{1n}}{\sum a_{nn}} \\ \frac{a_{21}}{\sum a_{n1}} & \frac{a_{22}}{\sum a_{n2}} & \cdots & \frac{a_{2n}}{\sum a_{nn}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{a_{n1}}{\sum a_{n1}} & \frac{a_{n2}}{\sum a_{n2}} & \cdots & \frac{a_{nn}}{\sum a_{nn}} \\ \sum \boldsymbol{a_{n1}} & \sum \boldsymbol{a_{n2}} & \cdots & \sum \boldsymbol{a_{nn}} \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} & \frac{\sum \mathbf{r_{1n}}}{\mathbf{n}} \\ r_{21} & r_{22} & \cdots & r_{2n} & \frac{\sum \boldsymbol{r_{2n}}}{\boldsymbol{n}} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ r_{n1} & r_{n2} & \cdots & r_{nn} & \frac{\sum \boldsymbol{r_{nn}}}{\boldsymbol{n}} \end{bmatrix} \quad \text{…(7)}$$

$$= \begin{bmatrix} r_{11} & r_{12} & \dots & r_{1n} & \boldsymbol{W_1} \\ r_{21} & r_{22} & \dots & r_{2n} & \boldsymbol{W_2} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ r_{n1} & r_{n2} & \dots & r_{nn} & \boldsymbol{W_n} \end{bmatrix} \quad \dots(8)$$

Step 4: Consistency Validation

After setting the priorities, the approved judgments must be consistent, and therefore attention should be paid to the quality of the decision and the consistency of the judgments made by the decision maker after a series of pairwise comparisons. Because (ideal) consistency is not easy to achieve because it is possible that inconsistencies will appear in each set of pairwise comparisons, this problem will be addressed. Scientifically, through the theory of Analytic Hierarchical Process (AHP) to measure the degree of consistency between marital judgments. If the degree of consistency is acceptable, the decision-making process can be completed, and vice versa. [22]

(AHP) provides a measure to verify the consistency of the judgments of pairwise comparisons by finding the value of the consistency ratio, so that if the resulting ratio is less than (0.1) it indicates that the judgments are consistent, otherwise they are inconsistent, and it includes calculating the consistency percentage according to the following steps [20].

1- Find the value of the eigenvector λ_n by multiplying the pairwise comparison matrix A by the row mean column W_n, as in the formula below:

$$a_{ij} = \frac{1}{a_{ij}}$$

$$= \frac{1}{\frac{w_i}{w_j}}$$

$$= \frac{w_j}{w_i} = 1 \quad \dots (9)$$

In the case of stability, then

$$\sum_{j=1}^{n} a_{ij} = \frac{w_j}{w_i} = n \qquad i = 1,2,3,\dots,n \quad \dots (10)$$

And by multiplying the above equation by W_i

$$\sum_{j=1}^{n} a_{ij}\, w_j = n w_i \qquad i = 1,2,3,\dots,n \quad \dots (11)$$

$$\text{Eigen Value} = A_{nxn} \, . \, W_{nx1} \quad \dots (12)$$

The matrix becomes as follows:

$$= \begin{bmatrix} 1 & a_{12} & \dots & a_{1n} \\ \frac{1}{a_{12}} & 1 & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{a_{1n}} & \frac{1}{a_{2n}} & \dots & 1 \end{bmatrix} . \begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ \vdots \\ W_n \end{bmatrix}$$

$$= \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \vdots \\ \lambda_n \end{bmatrix} \quad \dots (13)$$

2- Calculation of the consistency vector λ_{Max} by dividing the value of the eigenvector by the corresponding values of the mean of the rows, then sum the results of the division process and dividing them by the number of eigenvector values, as shown in the following relationship: [23]

$$\lambda_{Max} = \frac{1}{n} \sum_{i=1}^{n} \frac{\lambda n}{W_n} \quad \dots (14)$$

1- Calculation of the stability index CI It is calculated according to the following relationship

$$CI = (\lambda_{Max} - n) / (n - 1) \quad \dots (15)$$

2- Calculation of the stability ratio CR: It is calculated according to the following relationship:

$$CR = CI / RI \quad \dots (16)$$

RI: (Random Consistency Index) is the random consistency index of the pairwise comparison matrix. The value of RI depends on the number of elements that were compared with it. It is determined according to the table (2) of consistency index of the random judgment matrix. [24]

Table 2. Consistency index values for the random judgment matrix

n	RI	n	RI	N	**RI**
1	0	5	1.12	9	**1.45**
2	0	6	1.24	10	**1.49**
3	0.58	7	1.32	11	**1.51**
4	0.9	8	1.41	12	**1.58**

Step 5: Decision Making

The alternative that obtains the highest value is chosen from the matrix of relative standard weights, so that the decision is a reflection of the logical process of the human mind to sort the elements based on knowledge, experience and preference, and ensures consensus and a sound final decision. [25]

5. Integer Linear Programming Model

Some linear programming (LP) applications require a solution represented by integers, such as production problems. Therefore, integer linear programming (ILP) appeared so that all decision variables (Xi) are integer or binary (0,1) and the general formula of the model is as follows. [5], [26]

$$\text{Maximize } Z = \sum_{i=1}^{m}\sum_{j=1}^{n} W_{ij}\ X_{ij}$$

Subject to constraints

$$\sum_{i=1}^{m} X_{ij} \geq a_j \quad , j = 1,2,\dots,n \qquad \dots (17)$$

$$\sum_{j=1}^{n} X_{ij} \leq b_i \quad , i = 1,2,\dots,m$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} X_{ij} \leq S$$

$$Xij = 0 \text{ or } 1$$

6. Case Study

This study was applied in the General Company for Electrical and Electronic Industries in the Iraqi capital, Baghdad, and this company produces many electrical and electronic products in various fields, including (water pumps, lighting lamps, fans powered by electric and solar energy, electrical transformers, and others). In the beginning, the (AHP) method will be applied to determine the weights of the sample criteria and alternatives, and then those results will be the inputs of the correct linear programming model (ILP) to know the effect of those alternatives on the company's products.

- **Hierarchical structure**

After reviewing the officials of the company's departments, the main criteria and alternatives (objectives) that the company seeks to achieve were determined as shown in Figure (2), as well as in light of the results produced by the questionnaire forms, which were distributed to experts and specialists inside and outside the company to express their opinion on the importance of each criterion in relation to the other criteria Based on the importance degrees according to the (Saaty) scale. [16]

The significance value is determined for each criterion, and then the value of that importance is adopted in creating the pairwise comparison matrix that will be used to implement the research model.

The criteria

Productive efficiency and quality
Product competition
Costs required for production

The alternatives

Profit maximization
Production maximization
Reduce defects (damage)
Optimum use of energy and equipment
Inventory Management

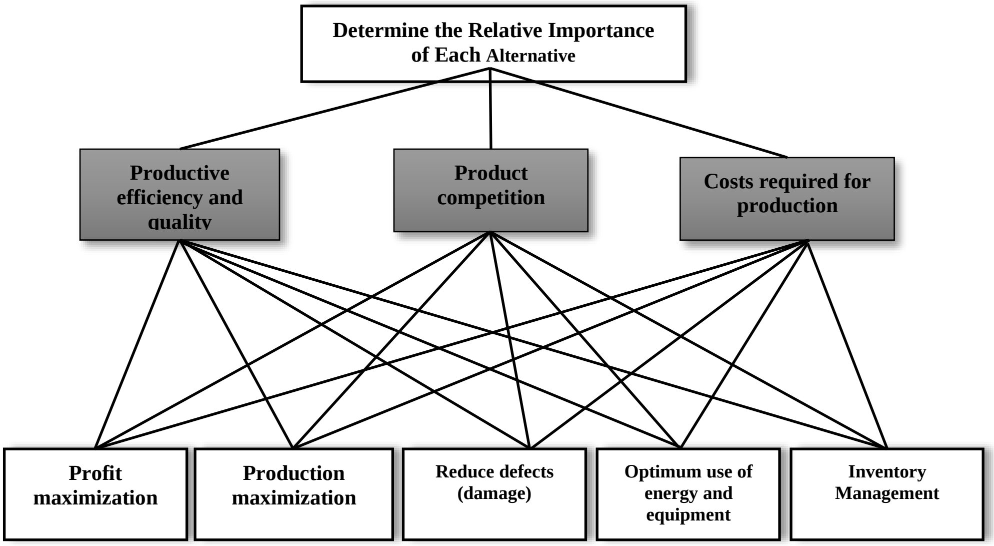


Figure 2. The hierarchy of the study sample

- **Pairwise Comparisons Matrices**

Pairwise comparison matrices for criteria and alternatives will be prepared based on the Analytic Hierarchical Process and based on the weights of the relative standards that we obtained from the questionnaire forms distributed over the research sample, and based on the degrees of importance of the (Saaty) scale, the importance value for each goal has been determined in the light of each criterion and for each criterion In light of the general objective by adopting the weights that obtained the highest number of preferences of the experts that they referred to in the questionnaire. and then adopting the importance values in creating the pairwise comparison matrix to be applied in the model by comparing the five alternatives in the light of each criterion separately, as follows:

A. Pairwise comparison matrix in the light of the criterion of production efficiency and quality

Table 3. Matrix of production efficiency and quality

Criteria 1	Alte.1	Alte.2	Alte.3	Alte.4	Alte. 5	wi
Alte .1	0.0760	0.0551	0.0456	0.1600	0.1315	0.0936
Alte. 2	0.2366	0.1531	0.3668	0.1649	0.3541	0.2551
Alte. 3	0.0970	0.0428	0.1556	0.0550	0.0534	0.0808
Alte. 4	0.1536	0.0770	0.2557	0.1650	0.0615	0.1426
Alte. 5	0.5459	0.4188	0.5542	0.6141	0.6172	0.5500

We calculate the value of the eigenvector λ_i by multiplying the Pairwise comparison matrix above in the row average column W, and then sum the elements of each row, we get the values of the eigenvector elements, as in the matrix in Table (4).

Table 4. Eigenvector

λi
0.582
0.6478
0.3482
0.5758
1.6625

B. Pairwise comparison matrix in light of the product competition criterion

Table 5. Product competition matrix

Criteria 2	Alte.1	Alte.2	Alte.3	Alte.4	Alte. 5	wi
Alte .1	0.2733	0.4153	0.3205	0.1214	0.1230	0.2507
Alte. 2	0.4354	0.2323	0.4595	0.4779	0.6272	0.4465
Alte. 3	0.1721	0.0468	0.1435	0.0636	0.1434	0.1139
Alte. 4	0.2191	0.2182	0.0468	0.0112	0.1554	0.1301
Alte. 5	0.2355	0.2125	0.1024	0.2656	0.1692	0.1970

C. Pairwise comparison matrix in light of the cost criterion required for production

Table 6. Production cost matrix

Criteria 3	Alte.1	Alte.2	Alte.3	Alte.4	Alte. 5	wi
Alte .1	0.2569	0.2876	0.1869	0.0750	0.0176	0.1648
Alte. 2	0.3270	0.3557	0.4187	0.1444	0.2418	0.2975
Alte. 3	0.2184	0.1385	0.0978	0.1527	0.2287	0.1672
Alte. 4	0.1765	0.1771	0.3419	0.1940	0.0699	0.1919
Alte. 5	0.5470	0.5512	0.4188	0.6208	0.4648	0.5205

D. Pair comparison matrix in light of the main objective

Table 7. Basic Goal matrix

Basic Goal	Criteria 1	Criteria 2	Criteria 3	Wi
Criteria 1	0.50	0.45	0.43	**0.46**
Criteria 2	0.30	0.35	0.33	**0.33**
Criteria 3	0.50	0.60	0.53	**0.54**

Now the general order of the decision alternatives' priorities (relative importance) will be determined, so the priority of each criterion will be dealt with as a weighting weight that reflects the importance of this criterion. Where we will get the general priority for each alternative by summing the product of wi for the three criteria in Table (7) with w_i for the five alternatives in Tables (3,5,6) to get:

Table 8. The relative importance of the sample alternatives

Alternative	relative importance
Profit maximization	0.2148
Production maximization	0.4253
Reduce defects (damage)	0.1650
Optimum use of energy and equipment	0.2122
Inventory Management	0.5991

The final results related to determining the main objective of the study sample (determining relative importance) among several alternatives, the importance of which is according to the results of the Analytic Hierarchical Process (AHP), and that inventory management was the most important goal of the sample; Therefore, it will have the first priority, followed by the four alternatives, respectively, which we will adopt in the weights of the mathematical model.

- **Application of the Integer Linear Programming (ILP) Model**

The process of formulating the ILP model includes relying on the results of the hierarchical analysis in maximizing the weight of the available alternatives and making the optimal decision as:

$$\text{Decision Variable } X_{i,j} = \begin{cases} 1 & \text{If Alternative i Impact to product j} \\ 0 & \text{Otherwise} \end{cases}$$

Where,

i= Alternative index, i=1,2,… m, m=number of Alternatives

j= Product index, j=1,2,…n, n= number of Products

$W_{i,j}$= Weightage of Alternative i for product j

a_j = Minimum requirement of Alternative for product j

b_i = Maximum number of products allocated to Alternative i

S = Total number of Alternative assignments needed for number of products

Table 9. products coding

Product Name	Code
Air Cooler	X_1
Solar Fan	X_2
Water Pump	X_3
LED	X_4
Air Compressor	X_5
Fire Extinguisher	X_6
Water Desalination System	X_7
Visual Monitoring System	X_8
Air Sanitizer	X_9
Distribution Transformer	X_{10}

Below is the formulation of the form and details of the solution:

MaxZ= $0.2148(X_{1,1}+X_{1,2}+X_{1,3}+X_{1,4}+X_{1,5}+X_{1,6}+X_{1,7}+X_{1,8}+X_{1,9}+X_{1,10})+0.4253(X_{2,1}+X_{2,2}+X_{2,3}+X_{2,4}+X_{2,5}+X_{2,6}+X_{2,7}+X_{2,8}+X_{2,9}+X_{2,10})+0.1650(X_{3,1}+X_{3,2}+X_{3,3}+X_{3,4}+X_{3,5}+X_{3,6}+X_{3,7}+X_{3,8}+X_{3,9}+X_{3,10})+0.2122(X_{4,1}+X_{4,2}+X_{4,3}+X_{4,4}+X_{4,5}+X_{4,6}+X_{4,7}+X_{4,8}+X_{4,9}+X_{4,10})+0.5991(X_{5,1}+X_{5,2}+X_{5,3}+X_{5,4}+X_{5,5}+X_{5,6}+X_{5,7}+X_{5,8}+X_{5,9}+X_{5,10})$ … (18)

Constraint 1: Choose at least one alternative for each product

$X_{1,1}+X_{2,1}+X_{3,1}+X_{4,1}+X_{5,1} \geq 1$

$X_{1,2}+X_{2,2}+X_{3,2}+X_{4,2}+X_{5,2} \geq 1$

$X_{1,3}+X_{2,3}+X_{3,3}+X_{4,3}+X_{5,3} \geq 1$

$X_{1,4}+X_{2,4}+X_{3,4}+X_{4,4}+X_{5,4} \geq 1$

$X_{1,5}+X_{2,5}+X_{3,5}+X_{4,5}+X_{5,5} \geq 1$

$X_{1,6}+X_{2,6}+X_{3,6}+X_{4,6}+X_{5,6} \geq 1$

$X_{1,7}+X_{2,7}+X_{3,7}+X_{4,7}+X_{5,7} \geq 1$

$X_{1,8}+X_{2,8}+X_{3,8}+X_{4,8}+X_{5,8} \geq 1$

$X_{1,9}+X_{2,9}+X_{3,9}+X_{4,9}+X_{5,9} \geq 1$

$X_{1,10}+X_{2,10}+X_{3,10}+X_{4,10}+X_{5,10} \geq 1$ …(19)

Constraint 2: Allocate all products to each alternative based on the sum of the weights of the alternatives

$X_{1,1}+X_{1,2}+X_{1,3}+X_{1,4}+X_{1,5}+X_{1,6}+X_{1,7}+X_{1,8}+X_{1,9}+X_{1,10} \leq 8$
$X_{2,1}+X_{2,2}+X_{2,3}+X_{2,4}+X_{2,5}+X_{2,6}+X_{2,7}+X_{2,8}+X_{2,9}+X_{2,10} \leq 12$
$X_{3,1}+X_{3,2}+X_{3,3}+X_{3,4}+X_{3,5}+X_{3,6}+X_{3,7}+X_{3,8}+X_{3,9}+X_{3,10} \leq 7$
$X_{4,1}+X_{4,2}+X_{4,3}+X_{4,4}+X_{4,5}+X_{4,6}+X_{4,7}+X_{4,8}+X_{4,9}+X_{4,10} \leq 10$
$X_{5,1}+X_{5,2}+X_{5,3}+X_{5,4}+X_{5,5}+X_{5,6}+X_{5,7}+X_{5,8}+X_{5,9}+X_{5,10} \leq 5$ …(20)

Constraint 3: Total benefits of alternatives allocated to all products

$X_{1,1}+X_{1,2}+X_{1,3}+X_{1,4}+X_{1,5}+X_{1,6}+X_{1,7}+X_{1,8}+X_{1,9}+X_{1,10}+X_{2,1}+X_{2,2}+X_{2,3}+X_{2,4}+X_{2,5}+X_{2,6}+X_{2,7}+X_{2,8}+X_{2,9}+X_{2,10}+$
$X_{3,1}+X_{3,2}+X_{3,3}+X_{3,4}+X_{3,5}+X_{3,6}+X_{3,7}+X_{3,8}+X_{3,9}+X_{3,10}+X_{4,1}+X_{4,2}+X_{4,3}+X_{4,4}+X_{4,5}+X_{4,6}+X_{4,7}+X_{4,8}+X_{4,9}+X_{4,10}+$
$X_{5,1}+X_{5,2}+X_{5,3}+X_{5,4}+X_{5,5}+X_{5,6}+X_{5,7}+X_{5,8}+X_{5,9}+X_{5,10} \leq 95$ …(21)

Constraint 4: Decision Variable are binary

$X_{1,1},X_{1,2},X_{1,3},X_{1,4},X_{1,5}, X_{1,6},X_{1,7},X_{1,8},X_{1,9},X_{1,10}, X_{2,1},X_{2,2},X_{2,3},X_{2,4},X_{2,5}, X_{2,6},X_{2,7},X_{2,8},X_{2,9},X_{2,10}, X_{3,1},X_{3,2},X_{3,3},X_{3,4},X_{3,5},$
$X_{3,6},X_{3,7},X_{3,8},X_{3,9},X_{3,10}, X_{4,1},X_{4,2},X_{4,3},X_{4,4},X_{4,5}, X_{4,6},X_{4,7},X_{4,8},X_{4,9},X_{4,10}, X_{5,1},X_{5,2},X_{5,3},X_{5,4},X_{5,5}, X_{5,6},X_{5,7},X_{5,8},X_{5,9},X_{5,10} = 0$ or 1 …(22)

Table 10. The impact of the alternatives on the company's products

Code	Product Name	Profit maximization	Production maximization	Reduce defects (damage)	Optimum use of energy and equipment	Inventory Management
X_1	1	1	1	1	1	1
X_2	1	1	1	1	1	1
X_3	1	1	1	1	1	0
X_4	1	1	0	1	1	1
X_5	1	1	1	1	1	1
X_6	1	1	1	1	1	1
X_7	1	1	1	1	1	1
X_8	1	1	1	1	1	1
X_9	0	1	0	1	1	0
X_{10}	0	1	0	1	1	0

The (ILP) model has been solved using (WINQSB) application, and the results are shown in Table (10) which includes the effect of the five alternatives on each of the company's products with maximizing the weights where the value of the objective function reached (Z=14.64).

7. Conclusion

Through this research, it was clarified how to use the Analytic Hierarchical Process (AHP) to determine the weights of the relative importance of criteria and alternatives on the one hand, and the application of a mathematical model (ILP) that helped in the process of determining and diagnosing the impact of those alternatives on the company's products without resorting to methods of estimation and estimation on the other hand. The model was very effective after being tested in the company under study, thus knowing the priorities of those alternatives for the decision maker in order to choose the optimal decision and achieve the maximum benefit. This model can also be applied in many other fields.

References

[1] H. A. Taha, *Operations Research An Introduction*, Tenth Edit. Pearson Education Limited, 2017.

[2] N. Bhushan and K. Rai, *Strategic Decision Making Applying the Analytic Hierarchy Process*. springer, 2004.

[3] B. Chandran, B. Golden, and E. Wasil, "Linear programming models for estimating weights in the analytic hierarchy process," *Computers & Operations Research*, vol. 32, no. 9, pp. 2235–2254, 2005.

[4] Z. Guan, Z. Jin, and B. Zou, "A multi-objective mixed-integer stochastic programming model for the vendor selection problem under multi-product purchases," *International journal of information and management sciences*, vol. 18, no. 3, p. 241, 2007.

[5] A. J. Rajan, "Application of Integer Linear Programming Model for Vendor Selection in a Two Stage Supply Chain," pp. 1–6,

2010.
[6] T. He, W. Ho, C. Lee Ka Man, and X. Xu, "A fuzzy AHP based integer linear programming model for the multi-criteria transshipment problem," *The International Journal of Logistics Management*, vol. 23, no. 1, pp. 159–179, 2012.
[7] G. Patel, G. D. Mjema, and K. M. Godwin, "Linear Programming Models For Estimating Weights In Analytic Hierarchy Process And For Optimization Of Human Resource Allocation," *International Journal of the Analytic Hierarchy Process*, vol. 8, no. 2, 2016.
[8] R. Al-Husain and R. Khorramshahgol, "Incorporating analytical hierarchy process and goal programming to design responsive and efficient supply chains," *Operations Research Perspectives*, vol. 7, p. 100149, 2020.
[9] M. E. Zerafati, A. Bozorgi-Amiri, A.-M. Golmohammadi, and F. Jolai, "A multi-objective mixed integer linear programming model proposed to optimize a supply chain network for microalgae-based biofuels and co-products: a case study in Iran," *Environmental Science and Pollution Research*, pp. 1–23, 2022.
[10] Y. Wang and K. Chin, "A linear programming approximation to the eigenvector method in the analytic hierarchy process," *Information Sciences*, vol. 181, no. 23, pp. 5240–5248, 2011, doi: 10.1016/j.ins.2011.07.009.
[11] E. H. Forman and M. A. Selly, *Decision by objectives:How to convince others that you are right*. World Scientific Publishing Co, 2001.
[12] A. Şehitoğlu, "Decision Support Model for Subcontractor Selection in Defense Industry Projects with Analytic Hierarchy Process and Integer Linear Programming," Hacettepe University, 2019.
[13] T. L. Saaty and L. G. Vargas, *Models, Methods, Concepts Sc Applications of the Analytic Hierarchy Proce*, Second Edi. International Series in Operations Research & Management Science,Springer, 2001.
[14] K. D. Young, "Application of the Analytic Hierarchy Process Optimization Algorithm in Best Management Practice Selection Master of Science," Virginia Polytechnic Institute, 2006.
[15] S. Nataraj, "Analytic Hierarchy Process As A Decision-Support System In The Petroleum Pipeline Industry," *Information Systems*, vol. VI, no. 2, pp. 16–21, 2005.
[16] T. L. Saaty and L. G. Vargas, *Models , Methods , Concepts & Applications of the Analytic Hierarchy Process*, Second Edi. International Series in Operations Research & Management Science,Springer, 2012.
[17] S. Oguztimur, "Why Fuzzy Analytic Hierarchy Process Approach For Transport," 2015.
[18] T. L. Saaty, "Decision making with the analytic hierarchy process," *Int.J.Services Sciences*, vol. 1, no. 1, pp. 83–98, 2008.
[19] S. K. Samanta, P. Jana, and S. Samajdar, "Selection and Sensitivity Analysis of Flare Piping Materials for the Onshore Hydrocarbon Industry.," *Journal of Production & Industrial Engineering (JPIE)*, vol. 4, no. 1, 2023.
[20] N. A. Baki and N. I. Norddin, "Application of Analytic Hierarchy Process for Selecting Best Student Application of Analytic Hierarchy Process for Selecting Best Student," *Journal of Applied Environmental and Biological Sciences*, vol. 7, no. 1, pp. 69–73, 2017.
[21] M. Kryvobokov, "Estimating the weights of location attributes with the Analytic Hierarchy Process in Donetsk , Ukraine," *Nordic Journal of Surveying and Real Estate Research*, vol. 2, no. 2, pp. 5–29, 2005.
[22] N. J. Mamat and J. K. Daniel, "Singular Value Decomposition Vs Duality Approach In Ahp : Time And Rank Comparisons In Faculty Member Selection," *ISAHP Hawaii*. p. 9, 2005.
[23] S. P. Kadu and P. S. Patil, "UAV (Unmanned Aerial Vehicle) for Agriculture Application-A State-of-the-Art Review.," *Journal of Production & Industrial Engineering (JPIE)*, vol. 2, no. 2, 2021.
[24] A. P. Edlabadkar and S. S. Chaudhari, "Literature Review on Optimization Techniques Used for Minimization of Casting.," *Journal of Production & Industrial Engineering (JPIE)*, vol. 4, no. 1, 2023.
[25] P. Bhattacharjee, R. K. Jana, and S. Bhattacharya, "Enhancement of Annual Profit of a Wind Farm Using Artificial Intelligence-Based Meta-heuristic Methodology.," *Journal of Production & Industrial Engineering (JPIE)*, vol. 3, no. 1, 2022.
[26] S. Kumar and S. R. Pulivarti, "Optimization of green sand mould process parameters using Taguchi approach in conjunction with Grey relational analysis.," *Journal of Production & Industrial Engineering (JPIE)*, vol. 2, no. 2, 2021.