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Abstract: The practical capacity of Human Powered Flywheel Motor (HPFM) to produce 3 to 5 Horse Power (H.P) in comparison to Motorized machine for shredding the monomer PVC Plastic waste for various applications gives it an advantage to operationalize it in rural sector. The present work is an extension of previously established dimensionless mathematical models using Buckingham Pi-theorem technique. The Current Paper represents the mathematical Model established for the response variable Torque required by the flywheel to shred the PVC Round Plastic pipe of 1 inch, 3/4 inch and 1/2 inch using the approximate Logarithmic Linearization technique. The mathematical model represented here has been generated after experimental results and it helps one to forecast the torque required by the HPFM unit to shred the PVC plastic waste of a particular kind efficiently so that it can be utilized in rural sector for various applications.

Keywords: HPFM, PVC Plastic, Logarithmic Linearization technique, torque

### 1. Introduction

The Shredding operation of PVC Plastic via Human Powered Flywheel Motor (HPFM) is a scientific research project which is being carried out to enhance the capacity of the machine to shred monomer PVC Plastic waste of any kind in collaboration with Human Scale Technologies Limited (HSTL) Bombay to promote the concept of green manufacturing. The shredded plastic of PVC are re-utilized in making electrical pipes, packaging material, agricultural pipes etc.[1] This Paper is an extension of the previously published work done where the dimensionless mathematical models for various response variables related to this machine has been established. In totality 5 independent dimensionless equations known as Pi'-1 to Pi'-5 have been established which can be seen as the function of the response variable Torque in Eq. (1) below in consecutive order which signifies namely.

- 1. Pie term related to the geometry of shredder blades  $\left(\frac{W_C*N_{C*}D*T_C}{L_C^3}\right)$ 2. Pie term related to the geometry of the raw material  $\left(\frac{LB_{rm}*OD_{rm}*WT_{rm}}{L_C^4}\right)$
- 3. Pie term related to the gear ratio (G)
- 4. Pie term related to the Material (Modulus of elasticity) & density of shredder blade and processed plastic -  $\left(\frac{M_{C}*\rho_{cm}*M_{P}}{L_{C}^{2}*\rho_{pm}^{3}}\right)$

5. Pie term related to the energy of the flywheel -  $\left(\frac{E_{\mathbf{F}}*\omega}{L_{c}^{3.5}*g^{0.5}*\rho_{pm}}\right)$ 

In the present research work we deal with the establishment of the mathematical model for one of the response variable Torque after experimentation. The equation for Torque is mentioned below [2] in Eq. (1).

The terminology for various variables in the above equation has been mentioned below in Table 1.

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Sr.no	Description of Variables	Types of Variables	Symbols
01	Energy of the flywheel	Independent	E <sub>F</sub>
02	Gear Ratio	Independent	G
03	Angular speed of the flywheel	Independent	ω
04	Acceleration due to gravity	Independent	g
05	Length of the cutting edge	Independent	L <sub>C</sub>
06	Width of the Cutting edge	Independent	W <sub>C</sub>
07	No. of cutting edges	Independent	N <sub>C</sub>
08	Diameter of Cutter	Independent	D
09	Thickness of cutter	Independent	Tc
10	Weight Density of cutter material	Independent	ρcm
11	Material of cutter (Modulus of Elasticity)	Independent	Mc
12	Length and breadth of Raw Material	Independent	LB <sub>rm</sub>
13	Outer diameter of the raw material	Independent	OD <sub>rm</sub>
14	Wall thickness of the raw material	Independent	WT <sub>rm</sub>
15	Weight Density of plastic material	Independent	ρ <sub>pm</sub>
16	Material of Plastic (Modulus of Elasticity)	Independent	Mp
17	Instantaneous Load Torque (T <sub>L</sub> )	Dependent	Td

Table 1: Terminologies for various variables mentioned in Eq. (1)

## 2. Methodology used for the calculation of Response Variable i.e. Torque

The experimental Set-up used for experimentation has been shown below in Fig 1, Fig 2 and Fig 3. The experimental set-up has three systems 1. Energy unit 2. The speed reducing spur gears of 60 teeth's, 80 teeth's and 100 teeth's which gives a reduction of 4.5, 6, 7.5 respectively and 3. The process unit i.e. the shredder blades.





Figure 3. Processing Unit (Shredder)

The equivalent Total Moment of inertia of the system for different gears used is given below	
Total Moment of Inertia $I_{Total} = I_F + I_{Shredder} = I_F + \frac{I_{Shredder}}{4.5^2}$ for small Gear with 60 teeth's	(2)
Total Moment of Inertia $I_{Total} = I_F + I_{Shredder} = I_F + \frac{I_{Shredder}}{6^2}$ for middle Gear with 80 teeth's	(3)
Total Moment of Inertia $I_{Total} = I_F + I_{Shredder} = I_F + \frac{I_{Shredder}}{4.5^2}$ for Large Gear with 100 teeth's	(4)
Where $I_F = Moment$ of Inertia for Flywheel and $I_{Shredder} = Moment$ of Inertia for the Shredder.	

Now to calculate the Instantaneous Load Torque [ $T_L$ ] coming on the flywheel after engagement of the clutch with process unit for a particular gear set (60, 80, or 100 teeth's) can be understood from the Fig 4 and the required sample calculations are explained below. Although terminal speed of 300 RPM and 500 RPM for each gear ratio (4.5, 6, 7.5) and size of pipe (1 inch, 3/4 inch & 1/2 inch) has been used for design of experimentation using Minitab software full factorial method. But considering an example of sample calculation for the flywheel the terminal speed of 500 RPM for the Gear set with 60 teeth's having a gear ratio of 4.5 for 1 inch PVC Round Pipe plastic shredding which runs for 09 sec's has been presented. Two inductive sensors have been employed to note down the readings built by Yogesh digital technologies as can be seen in Fig. 04. One inductive sensor on the flywheel to measure the terminal speed once it reaches 500 RPM and vice versa its reduction after its stored energy is utilized for the process unit. Another inductive sensor on the process unit to measure the variation in its speed once the energy of the flywheel is utilized for the process unit (plastic shredder) till the time one gets the output i.e. shredded plastic. The inductive sensor captures the speed with reference to time. The sample data can be seen in Table 2 below.

In Table 2. We can see that once the flywheel speed reaches near the speed of 500 RPM i.e. 495 RPM, the energy stored in the flywheel is given to process unit which shreds the PVC plastic pipe of 1 inch at 60 RPM and gradually the speed at both the terminals starts going down and we can see that when the flywheel speed reaches to 15 RPM the process unit RPM is zero which indicates that there is no output. This further reflects that the supply of energy is intermittent in nature for the machine but since the quality of the output products here in this case plastic shredding doesn't gets affected one can use this machine comfortably. From the data shared in the present table 2 it clearly shows that after each energization of the flywheel for the terminal speed of 500 rpm the process unit will run for 9 to 10 secs and the shredding operation will be carried out. The process is repeated till the availability of human energy to execute that. One can use this RPM to calculate the angular velocity of flywheel =  $\omega_f = \frac{2\pi N}{60}$  which can be seen in 4<sup>th</sup> and 7<sup>th</sup> column of table 2 for flywheel and process unit individually. The time mentioned here in the above table is from 88<sup>th</sup> sec to 96<sup>th</sup> sec which means effective shredding was for this period itself at 97<sup>th</sup> sec the process unit stopped irrespective of the flywheel

showing 15 RPM. The flywheel took 86 seconds to reach the terminal speed of 500 RPM so that it can be engaged to the process unit. As of now we want to quantify the response variable i.e. Torque w.r.t flywheel retardation hence we will plot a graph of angular velocity of flywheel w.r.t time as seen in Fig.5.

Time	Flywheel	RPM	Angular Velocity of flywheel (ω <sub>F</sub> )	Process Unit	RPM	Angular Velocity of shredder (ω <sub>Shred</sub> )	Time in Secs
31-03-2022							
11:30	N1(RPM)=	15	1.57	N2(RPM)=	0	0	97
31-03-2022 11:30	N1(RPM)=	45	4.71	N2(RPM)=	12	1.256	96
31-03-2022							
11:30	N1(RPM) =	75	7.85	N2(RPM) =	12	1.256	95
31-03-2022 11:30	N1(RPM)=	120	12.56	N2(RPM)=	24	2.512	94
31-03-2022	, , ,						
11:30	N1(RPM)=	195	20.41	N2(RPM)=	24	2.512	93
31-03-2022 11:30	N1(RPM)=	270	28.26	N2(RPM)=	24	2.512	92
31-03-2022							
11:30	N1(RPM)=	270	28.26	N2(RPM)=	24	2.512	91
31-03-2022							
11:30	N1(RPM)=	375	39.25	N2(RPM)=	24	2.512	90
31-03-2022							
11:30	N1(RPM)=	435	45.53	N2(RPM)=	36	3.768	89
31-03-2022							
11:30	N1(RPM) =	495	51.81	N2(RPM) =	60	6.28	88

Table 2. Speed of the flywheel and Process unit with respect to time





Now if one draws a tangent at different values of the plotted graph of Angular Velocity of flywheel retardation w.r.t time to calculate the value of  $\tan \theta = \frac{d}{dt} \omega_f$  as shown in Fig.6 below then one can calculate the value of Load Torque [T<sub>L</sub>] at each Point for a particular terminal speed engagement of the flywheel retardation as

For the present sample calculation the value of load Torque ( $T_L$ ) at each point of the flywheel retardation at 500 RPM terminal speed can be seen in Table 3 below.



**Figure 6.** Represents how to calculate the tan  $\theta$  i.e.  $\frac{d}{dt}\omega_f$ 

Sr. No	Total Moment of Inertia (I <sub>Total</sub> )	Angle	$\tan  heta$	Load-Torque- (T <sub>L</sub> ) in (N-m)	Time in Secs
1	2.777	0	0.000	0.000	97
2	2.777	52	1.280	3.554	96
3	2.777	50	1.192	3.309	95
4	2.777	46	1.036	2.876	94
5	2.777	40	0.839	2.330	93
6	2.777	35	0.700	1.944	92
7	2.777	35	0.700	1.944	91
8	2.777	30	0.577	1.603	90
9	2.777	29	0.554	1.539	89
10	2.777	22	0.404	1.122	88

Table 3 Value of Load Torque (T<sub>L</sub>) for 500 RPM Small Gear- Shredder-II- 1 inch PVC Round Pipe

Moreover if we see the response variable for Resistive Torque in dimensionless equation form then it is represented as  $\pi_{D1=} \frac{T_L}{L_c^4 * \rho_{pm}}$  in Eq. (1) where the Load Torque (T<sub>L</sub>) has been divided by the length of the cutting edge of the used shredder blades and Weight Density of plastic material. However the later part of the equation is the function of 05 independent Pi terms in consecutive order as

$$\left\{ \left( \frac{W_{C^*N_{C^*}R_C^*D^*T_C}}{L_C^3} \right) \left( \frac{LB_{rm}^*OD_{rm}^*WT_{rm}}{L_C^4} \right) (G) \left( \frac{M_C^*\rho_{cm}^*M_P}{L_C^{2*}\rho_{pm}^3} \right) \left( \frac{E_{F^*\omega}}{L_c^{3.5}*g^{0.5*}\rho_{pm}} \right) \right\}.$$

The significance of each independent Pi term has already been explained above. The value of Load Torque ( $T_L$ ) for each point in the graph of Fig.6 for 500 RPM Small Gear- Shredder-II- 1 inch PVC Round Pipe used as sample example for the case study has been calculated in table 3 above, but we still need to calculate the dimensionless value of Restive Torque as per Eq. (1) and the Value of all independent Pi terms (Pi'-1 to Pi'-5) which has been calculated in Table 4 and the final values of response variable Torque and input independent parameters are converted into Log values as shown in Table 5 below to plot their 2D graphs:-

Sr. No	Pi'-1	Pi'-2	Pi'-3	Pi'-4	Pi'-5	Total Pi' (Multiplication of all Pi' terms)
1	44.237	6.0041	4.5	1.2E+17	2795701.779	4.01667E+26
2	44.237	6.0041	4.5	1.2E+17	1897330.625	2.72596E+26
3	44.237	6.0041	4.5	1.2E+17	1215539.424	1.7464E+26
4	44.237	6.0041	4.5	1.2E+17	453697.6591	6.51842E+25
5	44.237	6.0041	4.5	1.2E+17	453697.6591	6.51842E+25
6	44.237	6.0041	4.5	1.2E+17	170914.5674	2.45558E+25
7	44.237	6.0041	4.5	1.2E+17	39830.79586	5.72262E+24
8	44.237	6.0041	4.5	1.2E+17	9724.315395	1.39712E+24
9	44.237	6.0041	4.5	1.2E+17	2100.452125	3.01779E+23

 Table 4 Calculation for Independent Pi' terms 500 RPM Small Gear- Shredder-II- 1 inch PVC Round Pipe

Sr. No	Load Torque- T <sub>L</sub> in (N- m)	Time in Sec's	Lc4*density of the Plastic (p <sub>pm</sub> )	Resisting Torque (π <sub>D1</sub> ) (dimensionles s)	Total Pi' (all 05 independent Variables)	Log (π <sub>D1</sub> )	Log Total Pi
01	0.000	97	0.0034863	0.0000	0.00E+00	00	00
02	3.554	96	0.0034863	1019.5330	3.01779E+23	6.9271	54.06398
03	3.309	95	0.0034863	949.2871	1.39712E+24	6.855711	55.59646
04	2.876	94	0.0034863	824.8480	5.72262E+24	6.715199	57.00647
05	2.330	93	0.0034863	668.3819	2.45558E+25	6.50486	58.46299
06	1.944	92	0.0034863	557.7479	6.51842E+25	6.323907	59.43926
07	1.944	91	0.0034863	557.7479	6.51842E+25	6.323907	59.43926
08	1.603	90	0.0034863	459.8863	1.7464E+26	6.130979	60.42477
09	1.539	89	0.0034863	441.5329	2.72596E+26	6.090253	60.87003
10	1.122	88	0.0034863	321.8257	4.01667E+26	5.77401	61.25767

Table 5 Represents the calculated values of Resistive Torque (Response Variable) and Total Independent Pi term

# **3.** Establishment of Mathematical Model using approximate Logarithmic Linearization technique for the sample calculation of 500 RPM Small Gear- Shredder-II- 1 inch PVC Round Pipe

Logarithmic Linearization Technique is an approximate mathematical technique to linearly represent a non-linear relationship. In our current research the input shaft of the processing unit gets accelerated and achieves maximum speed, and then due to the resistive torque offered by the processing unit gets decelerated. Thus, the input shaft is in a transient state of motion. The plastic shredder unit process system comes under the category of highly complex discrete time dynamic problem. These kinds of system usually have nonlinear difference equations as their solutions. For such issues, there is typically no closed-form solution. Therefore, we are forced to use numerical and/or approximation techniques such as logarithmic linearization to generate a linear. Equation of the form  $y = K(X)^n$ . The measure of goodness of fit  $(R^2)$  for the equations will validate the experimental data obtained. As per the dimensional analysis, five independent  $\pi$ terms and two dependent  $\pi$  term for Torque for 03 different sizes of PVC conduit pipe (1/2 inch, 3/4 inch, 1 inch) for shredder 01 & Shredder 02 have been developed. All the five independent  $\pi$  terms are multiplied to form a single independent  $\pi$  term known as Total  $\pi$  of independent parameter and is converted into its log value and plotted on X-axis for each model. Similarly the log of dependent parameter values for each model is plotted on Y-axis. Hence a graph is plotted between X-Y to find a linear relation between the inputs i.e. independent parameters and the output i.e. the dependent parameter in terms of logarithmic linear equation as mentioned below in Eq. (6). The converted sample log values for (independent and dependent parameter) of one such experimental reading can be seen in table 5 (column 7 & Column 8) above along with the plotted graph of Logarithmic Linearization equation in terms of  $y = 3.471*10^{6}(x)^{-0.148}$ Fig 12 below.

$y = K(X)^n$	(6)
Taking log on both sides	
$Log y = log K + log X^{n}$	(7)
Log y = log K + n log X	(8)
$Log y = n \log X + \log K$	(9)
If we consider $y = \text{Log } y$ , $m=n$ , $x = \log X$ , $c = \log K$	
y = mx + c which is actually a equation of straight line.	(10)

The logarithmic linear equation for each set of experimental readings are formed for Torque is plotted on Y-axis against the total  $\pi$  of independent parameters (input) which is plotted on X-axis. The measure of goodness of fit ( $R^2$ ) for each particular graph is also obtained to validate the relationship between input data and output data. The graphs and the value of ( $R^2$ ) for the entire experimental output obtained has been created using "**Origin-07**<sup>th</sup> **Version software for Graph & Analysis**". With Similar calculation equations for other terminal speed of the flywheel along with pipe size variation the Logarithmic Linearization 2-D graphs along with their individual equations are shown below:-



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### 4. Results and Discussion

- 1. The Logarithmic values for Instantaneous Load Torque (dependent Variable) on y axis and Total Pi' (independent Variables) on x-axis has been obtained for different terminal speeds of the flywheel, for different gear, Pipe Size and shredder which can be depicted from the graphs from Fig 7 to Fig 42.
- 2. The graph for the Instantaneous Load Torque (dependent Variable) and Total Pi' (independent Variable) has been plotted to obtain the equation in logarithmic linearization form  $y = K(X)^n$  for each individual

experimentation conducted within the selected test envelope. The sample calculations have been shown in table 02 to table 05 above along with the logarithmic linearization graph in Fig 12 for the same sample calculation i.e. 500 RPM- Small Gear- 1 inch PVC Pipe-Shredder-II.

3. The measure of goodness of fit (R<sup>2</sup>) for each particular graph including the sample experimental calculation graph Fig. 12 for all values of Instantaneous load Torque (output) with respect to Total Pi' terms (input) is found to be equal to or more than 0.90.

#### 5. Conclusion

The formulation of theoretical dimensionless mathematical Logarithmic Linearization equations has been developed for the response (dependent) variable for the plastic shredding operation through the HPFM system by analyzing the various parameters of machine, shredder, and plastic. This mathematical model is the representation of entire experimentation results which predicts the direct relation between the dependent parameters of the plastic shredding process, i.e. instantaneous load torque and independent parameters in terms of Total Pi'. The measure of goodness of fit ( $R^2$ ) for each particular graph obtained above is found to be greater than equal to 0.90 which clearly validates the relationship between input data and output data. The deduced set of equations obtained above From Fig. 7 to Fig. 42 obtained in terms of logarithmic linearization equation will generate the best research knowledge for the plastic shredding operation through the HPFM system within the test envelope under which the experimentation has been conducted. Using these equations one can generate the design data for torque calculation according to the requirement. The experimental results obtained using these equations will generate an approximate generalized Experimental data-based logarithmic linearization mathematical model for the plastic shredding process through HPFM. Overall the Logarithmic linearization technique endorses the output received for the present experimentation with the selected input variables.

The scope of this research work is very wide and the idea to convert the HPFM machine into a mechanical flywheel battery of reduced weight and size in comparison to the present m/c can tremendously ramp up the efficiency and the application scope. The thought on this will be a novel work.

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