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Application of 2^4 Factorial Experiment in Analyzing the Effect of Four Factors of Cracks on Components for Jet Turbine Aircraft Engines

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Abstract - This study investigates the effect of four factors: pouring temperature (A), titanium content (B), heat treatment method (C) and amount of grain refiner (D) as well as the interaction of pouring temperature × titanium content (AB), pouring temperature × heat treatment method (AC), titanium content × heat treatment (BC), heat treatment × amount of grain refiner (CD), pouring temperature × titanium content × heat treatment method (ABC) and pouring temperature × titanium content × heat treatment method × amount of grain refiner (ABCD) on the length of crack in the components of a jet turbine engine. This was made possible by the application of the 2^4 Factorial Design to analyse data which is on nickel-titanium used to create components for a jet turbine engine. The results revealed that the four factors take a substantial consequence on cracking in the parts of a jet turbine engine. The significant effect of the factors on cracking was further confirmed by a multiple linear regression model. Two multiple comparison methods were employed in the study to compare all pairs of means involving the main factors effects in the data, the reason for which is to determine which of the main factor effect affects variability in cracking and these multiple comparison techniques are the Duncan's multiple range (DMR) test as well as the least significance difference (LSD) method. The result of the multiple comparison revealed that the least significance difference (LSD) method produced the same outcome with the Duncan's multiple range (DMR) method. The analysis in this article were analyzed and executed with the Statistical Package for Social Sciences *(SPSS)* software and Microsoft Excel.

Keywords - factorial, cracking, regression, factor effects, multiple comparison, error

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1. Introduction

Apparatuses for aircraft turbine jet engines are made from nickel-titanium alloy. Cracking is a possibly grave problem in the finishing portion and may cause an irreparable catastrophe. However, titanium has long been regarded as taking a necessary stability of properties for applications near the anterior culmination of the gas turbine engine, that is, the fan discs/blades, compressor discs/blades, besides additional minor components. Titanium has a density of $4.5g/cm^3$. This means that the density does not differ considerably in alloys considered for aerospace uses, separately from inadequate number of alloys such as T_i811, which makes it lower than nickel and steel alloys, but higher than aluminum. Titanium is allotropic characterized by a hexagonal close-packed (HCP) lattice (α phase) steady to a temperature of 882^0C which changes to a body centered cubic (BCC) lattice (β phase) above 882^0C. The α and β phases are stabilized by the alloying elements. For example, Vanadium (V), Molybdenum (Mo) and Chromium (Cr) stabilizes the β phase, while Silicon (Sn) and Aluminum (Al) stabilizes the α phase; this shows that the makeover temperature can be changed, and consequently the magnitudes of each segment remaining at room temperature can be different. The morphology of these s with secondary α configurations (emerging from the β → α phase transformation). This process permits the growth of a variety of bimodal microstructures that produce titanium alloys with essential power and tolerates advance modification of properties over several heat treatment as well as treating systems [1].

Factorial designs are broadly employed in experiments encompassing numerous factors where it is required to study the combined effect of the factors in a response. A factorial experiment may have n factors each at 2, 3, 4,…, levels. The n factor factorial experiment both at two stages is paramount in research work, because it forms the foundation of other factorial designs of substantial value. "Reference [2]" asserts that a complete replicate of such a design requires $2\times2\times\ldots\times2 = 2^n$ observations and is called a 2^n factorial design. Consequently, these designs are widely used in factor screening experiments. In this study, we employed a dual replicate of a design with n factors both at two stages where n = 4 that is, a distinct duplicate of the 2^4 design.

"Reference [3]"explained that a factorial design is a type of research procedure that permits the study of the main and interaction effects between two or more independent variables as well as one or more outcome variable(s). "Reference [4]" also argued that factorial designs epitomizes the true commencement of contemporary behavioral research and have produced a major paradigm shift in the manner social researchers hypothesize their research enquiries in order to deliver objective conclusions.

This study employed data from a test run by the parts producer of the jet turbine engine to regulate the consequence of four factors on cracks. The four factors are pouring temperature (A), titanium content (B), heat treatment method (C), and amount of grain refiner used (D) [2] .The study is also centered on the following objectives (i) To employ two replicates of a 2^4 factorial design to estimate the four factors effects on cracks and to determine which factor effect seem to be big (ii) To comport a technique of analysis of variance in order to determine if one of the features affects cracking at $\alpha = 0.05$ significant level (iii) To develop a model of regression that would be employed to forecast crack dimension as a function of the substantial leading effects and interactions identified in part (ii) and (iv) To evaluate the deviations in the investigation and to determine if there is an indication that any of the features disturb the capriciousness in cracking.

The rest of this study is structured as follows. In Section II, we present the methodology of the study which deals with the general 2^n factorial design. In Section III, we dealt with the model of regression for forecasting crack dimension as a function of the weighty leading interactions and effects identified in the ANOVA technique. In section IV, we present the analysis of the data using the 2^4 factorial design with the regression model as well as the response surface. In Section V, we present the conclusion and direction for upcoming studies in factorial experiments.

2. The 2^n Factorial Design

The general 2^n factorial experiment characterizes merely a distinctive instance of the general P^n factorial design, that is, n factors with P levels each. However, we shall use the general 2^n case to lay the basis for the discussion of the 2^4 case by presenting suitable notation and mathematics appropriate for generalization in the article. The observations in a 2^4 factorial design can be described by the model given as

$$y_{ijklm}=\mu + \alpha_i + \beta_j + \gamma_k + \lambda_l + (\alpha\beta)_{ij} + (\alpha\gamma)_{ik} + (\alpha\lambda)_{il} + (\beta\gamma)_{jk} + (\beta\lambda)_{jl} + (\gamma\lambda)_{kl} + (\alpha\beta\gamma)_{ijk} + (\alpha\beta\lambda)_{ijl} + (\beta\gamma\lambda)_{jkl} + (\alpha\gamma\lambda)_{ikl} + (\alpha\beta\lambda\gamma)_{ijkl} + \varepsilon_{ijklm} \quad (1)$$

$i =1,2,\ldots,a$; $j=1,2,\ldots,b$; $k =1,2,\ldots,c$; $l =1,2,\ldots,d$ and $m =1, 2\ldots n$

where μ is the overall mean effect, α_i is the effect of the i^{it} level of factor *A*, β_j is the effect of the j^{th} level of factor *B*, γ_k is the effect of the k^{th} level of factor C, λ_l is the effect of the l^{th} level of factor D,$(\alpha\beta)_{ij}$ is the effect of the interaction between α_i and β_j, $(\alpha\gamma)_{ik}$ is the effect of the interaction between α_i and γ_k,$(\alpha\lambda)_{il}$ is the effect of the interaction between α_i and λ_l,$(\beta\gamma)_{jk}$ is the effect of the interaction between β_j and γ_k, $(\beta\lambda)_{jl}$ is the effect of the interaction between β_j and λ_l, $(\gamma\lambda)_{kl}$, is the effect of the interaction between γ_k and λ_l, $(\alpha\beta\gamma)_{ijk}$ is the effect of the interaction between α_i,β_j and γ_k,$(\alpha\beta\gamma)_{ijl}$ is the effect of the interaction between α_i,β_j and λ_l, $(\beta\gamma\lambda)_{jkl}$ is the effect of the interaction between β_j,γ_k, and λ_l, $(\alpha\gamma\lambda)_{ikl}$ is the effect of the interaction between α_i,γ_k and λ_l,$(\alpha\beta\gamma\lambda)_{ijkl}$ is the effect of the interaction between α_i, β_j,γ_k and λ_l, and ε_{ijklm} is the random error component.

In this study, four factors are of interest each at two levels and this gives the reason for the 2^4 factorial design such that the treatment combination emerging from the design are 16 in number as can be seen in the model equation (1). Also, the 16 treatment combinations can be viewed as $2^4 =2\times2\times2\times2=16$. We write these treatment combinations in standard order as (1), a, b, ab, c, ac, bc, abc, d, ad, bd, abd, cd, acd, bcd, abcd. We used the "+" and "-" orthogonal coding to represent the low and high levels of the factors and we present the sixteen runs in the 2^4 design as in Table I. In Table I, a representation of algebraic signs with the sixteen treatment combinations are displayed. These sixteen runs with the algebraic signs are used to form what we called contrast constant that would be employed to estimate the main effect of the factors. Sum of squares for the effect is computed from the contrast constant and each effect has a corresponding single degree of freedom contrast. In the 2^4 factorial design with n replicates, the sum of squares for any effect is given by

$$SS = (\text{contrast})^2/16n \quad (2)$$

The effect of the factors only in the 2^4 design with n replicates is given by the relation (3).

$$\text{Effect of factor} = (\text{contrast})/8n \tag{3}$$

The remaining factor effects are computed in the same manner as in equation (4). The sum of squares for factor A effect is computed by equation (5). Thus, the sum of squares for factor A is given by

$$SS_A = \frac{(a-(1)+b+ab-c+ac-bc+abc-d+ad-bd+abd-cd+acd-bcd+abcd)}{16n} \tag{5}$$

The remaining sum of squares factor effects are computed in the same manner as in equation (5). The total sum of squares is computed by

$$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l-1}^{d}\sum_{m-1}^{n} y_{ijkl}^2 - \frac{y_{.....}^2}{16n} \tag{6}$$

The sum of squares error component is obtained by subtracting the sum of squares for all the factor effects and interaction effects from the total sum of squares component, which is given by [5].

$$\begin{aligned} SS_E &= SS_A - SS_B - SS_{AB} - SS_C - SS_{AC} \\ &SS_{BC} - SS_{ABC} - SS_D - SS_{AD} - SS_{BD} \\ &- SS_{ABD} - SS_{CD} - SS_{ACD} - SS_{BCD} - SS_{ABCD} \end{aligned} \tag{7}$$

The technique of examination of variance otherwise known with the acronym ANOVA is employed in this study to confirm the magnitude of the feature effects. The procedure for the ANOVA technique for the 2^4 factorial design is displayed in Table 2.

3. The Regression Model for Predicting the Factor Effects in a 2^4 Factorial Design

The term regression is employed to describe statistical relations amid variables. A regression model is a formal means of stating the two necessary components of a statistical relation. The two crucial ingredients of a statistical relation has to do with the propensity of the response variable Y to differ with the predictor variable X in a logical manner [6]. In this study, we employed the general linear regression model, with normal error terms, simply in terms of X variables. The general linear regression equation is given by equation (8).

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \ldots + \beta_{q-1} X_{i,\,p-1} + \varepsilon_i \tag{8}$$

where $\beta_0, \beta_1 \ldots \beta_{q-1}$ are parameters. $X_{i1} \ldots X_{i,\,q-1}$ are known parameters, ε_i are independent N $(0, \partial^2)$, i=1,…, n.

The response function for regression function (8) given by equation (9), since E $\{\varepsilon_i\} = 0$

$$E[Y_i] = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \ldots + \beta_{q-1} X_{i,p-1} \tag{9}$$

Table 1. Signs for Effects in the 2^4 Factorial Design

	A	B	AB	C	AC
(1)	-	-	+	-	+
a	+	-	-	-	-
b	-	+	-	-	+
ab	+	+	+	-	-
c	-	-	+	+	-
ac	+	-	-	+	+
bc	-	+	-	+	-
abc	+	+	+	+	+
d	-	-	+	-	+
ad	+	-	-	-	-
bd	-	+	-	-	+
abd	+	+	+	-	-
cd	-	-	+	+	-
acd	+	-	-	+	+

bcd	-	+	-	+	-
abcd	+	+	+	+	+
	BC	ABC	D	AD	BD
(1)	+	-	-	+	+
a	+	+	-	-	+
b	-	+	-	+	-
ab	-	-	-	-	-
c	-	+	-	+	+
ac	-	-	-	-	+
bc	+	-	-	+	-
abc	+	+	-	-	-
d	+	-	+	-	-
ad	+	+	+	+	-
bd	-	+	+	-	+
abd	-	-	+	+	+
cd	-	+	+	-	-
acd	-	-	+	+	-
bcd	+	-	+	-	+
abcd	+	+	+	+	+
	ABD	CD	ACD	BCD	ABCD
(1)	-	+	-	-	+
a	+	+	+	-	-
b	+	+	-	+	-
ab	-	+	+	+	+
c	-	-	+	+	-
ac	+	-	-	+	+
bc	+	-	+	-	+
abc	-	-	-	-	-
d	+	-	+	+	-
ad	-	-	-	+	+
bd	-	-	+	-	+
abd	+	-	-	-	-
cd	+	+	-	-	+
acd	-	+	+	-	-
bcd	-	+	-	+	-
abcd	+	+	+	+	+

In general, the variables $X_1,\ldots,X_{q-1}$ in a regression model do not need to represent different predictor variables. Thus, the general linear regression model with normal error terms implies that the observations Yi are independent normal variables, with mean $E\{Y_i\}$ as given by (9) and with constant variance ∂^2.

In this article, the coefficients of the regression model (8) are estimated with respect to the 2^4 factorial design. Here, the regression coefficients β_1, β_2, β_3, $\beta_4,\ldots$, βq-1, are one-half the corresponding factor effects. The regression coefficient is one – half the effect estimate because a regression coefficient measures the effect of a one-unit change in x on the mean of y and the effect is based on a two-unit change from (-1) low level of a factor to (+1) high level of a factor. This technique of estimating the regression coefficients results to least squares technique for parameter estimates.

Table 2. Analysis of Variance for the 2^4 Factorial Design

Source of variation	Sum of squares	Degrees of freedom	Mean square	F_0
A	SS_A	a-1	$SS_A/a-1$	MS_A/MS_E
B	SS_B	b-1	$SS_B/b-1$	MS_B/MS_E
C	SS_C	c-1	$SS_C/c-1$	MS_C/MS_E
D	SS_D	d-1	$SS_D/d-1$	MS_D/MS_E
AB	SS_{AB}	(a-1)(b-1)	$SS_{AB}/(a-1)(b-1)$	MS_{AB}/MS_E
AC	SS_{AC}	(a-1)(c-1)	$SS_{AC}/(a-1)(c-1)$	MS_{AC}/MS_E
AD	SS_{AD}	(a-1)(b-1)	$SS_{AD}/(a-1)(d-1)$	MS_{AD}/MS_E
BC	SS_{BC}	(b-1)(c-1)	$SS_{BC}/(b-1)(c-1)$	MS_{BC}/MS_E
BD	SS_{BD}	(a-1)(b-1)	$SS_{BD}/(b-1)(d-1)$	MS_{BD}/MS_E
CD	SS_{CD}	(c-1)(d-1)	$SS_{CD}/(c-1)(d-1)$	MS_{CD}/MS_E
ABC	SS_{ABC}	(a-1)(b-1)(c-1)	$SS_{ABC}/(a-1)(b-1)(c-1)$	MS_{ABC}/MS_E
ABD	SS_{ABD}	(a-1)(b-1)(d-1)	$SS_{ABD}/(a-1)(b-1)(d-1)$	MS_{ABD}/MS_E
ACD	SS_{ACD}	(a-1)(c-1)(d-1)	$SS_{ACD}/(a-1)(d-1)(c-1)$	MS_{ACD}/MS_E
BCD	SS_{BCD}	(b-1)(c-1)(d-1)	$SS_{BCD}/(b-1)(d-1)(c-1)$	MS_{BCD}/MS_E
ABCD	SS_{ABCD}	(a-1)(b-1)(c-1)(d-1)	$SS_{ABCD}/(a-1)(b-1)(d-1)(c-1)$	MS_{ABCD}/MS_E
Error	SS_E	abcd(n-1)	$SS_E/abcd(n-1)$	
Total	SS_T	abcdn-1		

4. Data Analysis

In this section, we present the analysis of the 2^4 factorial design on the nickel – titanium data employed in manufacturing mechanisms for aircraft turbine jet engines. Since the2^4 factorial design is characterized by four factors each at two levels, such that the two levels are signifying the high and low levels of the factors. The high level of a factor is represented by (+1), whereas the low level of a factor is represented by (-1). From the data in Table III, the first factor is pouring temperature and is represented by capital letter A, the second factor is titanium content and is represented by capital letter B, the third factor is heat treatment method and is represented by capital letter C, and while the fourth factor is the quantity of particle refiner used and is represented by capital letter D. The data is in two duplicates of a 2^4 factorial design with the length of the crack measured in (mm × 10^{-2}) as prompted in a sample form exposed to a typical examination. The data is presented in Table III.

The main effects of all the four factors considered here with their interaction effects and their respective sum of squares are computed from equations (4), (5), (6) and (7), For example, the main effect of factor $A$is computed as follows.

$$A= (a-(1) +b+ab-c+ac-bc+abc-d+ad-bd+abd-cd+acd-bcd+abcd) /8n$$

$$= (-13.413+29.926-23.724+35.088-20.55+8.466-18.61+23.36-17.512+33.919-27.534+39.463-24.183+12.029 - 22.125+30.706)/8 \times 2$$

$$= (48.302)/16 = 3.0189$$

Therefore, the main effect of factor A is 3.0189. The main effects of other factors with their interaction effects are computed in the same manner with that of factor A. The sum of squares of factor A is computed as follows.

$$SS_A = (a-(1) +b+ab-c+ac-bc+abc-d+ad-bd+abd-cd+acd-bcd+abcd)^2/16n$$

$$= (-13.413+29.926-23.724+35.088-20.55+8.466-18.61+23.36-17.512+33.919-27.534+39.463 -24.183+12.029-22.125+30.706)^2/16 \times 2$$

$$= (48.302)^2/16 =72.908850125$$

Therefore, the sum of squares for factor A=72.908850125. The sum squares for the remaining factors and their respective interaction sum of squares are computed in the same manner with the sum of squares factor A. Sum of squares totals (SS_T) and sum of squares error (SS_T) are respectively computed from equations (6) and (7) . A summary of the factor effect estimates and sums of squares as well as the respective percentage contribution of the factor effects for the 2^4 factorial design for the data in Table III is presented in Table IV. In performing the 2^4 factorial design analysis that produced the results in Table IV, we employed the Statistical Package for Social Sciences *(SPSS)* software for the analysis. The Flow Chart that depicts the steps taken in *SPSS* analysis is shown in Fig. 2. The statement of hypothesis for this experiment is stated as follows.

$$H_0: Factor_A = Factor_B = Factor_C = Factor_D$$

$$H_a: Factor_A \neq Factor_B \neq Factor_C \neq Factor_D$$

The null hypothesis is represented by H_0 which is a statement of no significant effect. The statement implies that the factors (pouring temperature: A, titanium content: B, heat treatment method: C, and amount of grain refiner: D have no significant effect on the length of crack on components of jet turbine engines. The alternative hypothesis is represented by H_a which is a statement of the existence of significant variation between the factors in the experiment. The statement implies that the factors in the experiment have significant effect on the length of crack on components of jet turbine engines. The analysis of variance (ANOVA) table for the 2^4 factorial design in this experiment is presented in Table V. The normal probability plot of the effects for the 2^4 factorial design is presented in Fig. 1.

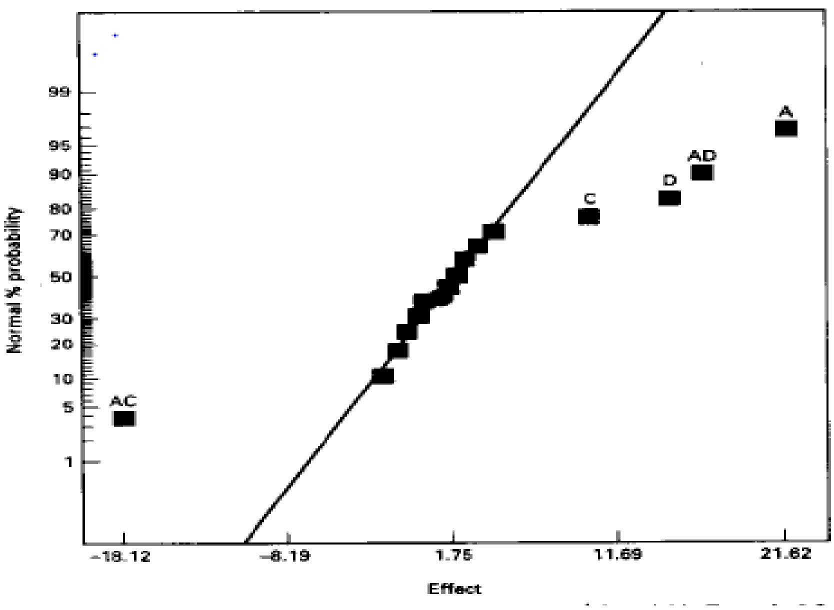

Figure 1. Normal Probability Plot of the Effects for the 2^4 Factorial Design

Table 4 summarizes the effect estimates and sum of squares. Also, the percentage contributions in the third column of Table 4 measures the percentage contribution of each model term relative to the total sum of squares. The input of percentage to the 2^4 factorial model terms depicts an uneven but then again an active conductor to the significance of the terms. In Table 4 again, one would observe that the main effect of B (titanium) contributes 22.1489% to the process and it is the highest among the main effects. This trend reveals that the factor B (titanium content) effect actually governs the course, thereby accounting for over 22% of the total variability. In Table IV, one would also observe that the interaction effect of AC (pouring temperature × heat treatment method) contributes 22.505% to the process and it is the highest among the interaction effects. This trend reveals that the AC (pouring temperature × heat treatment method) interaction factor effect really dominates the remaining effects of interaction in the process. The AC (pouring temperature × heat treatment method) interaction effect accounts for over 22% of the total variability with respect to the interaction effects. However, comparing the percentage contributions of the factor B (titanium content) main effect and the AC (pouring temperature × heat treatment method) interaction effect, one would observe that `the contribution of the AC (pouring temperature × heat treatment method) interaction effect is slightly above the contribution of the factor main effect of (titanium content) to the process. This shows that holistically the AC (pouring temperature × heat treatment method) gives the highest contribution of total variability to the process. Also, the percentage contributions of the main effects of A (pouring temperature), C (heat treatment method), D (grain refiner) account respectively for about 13%, 18%, 5%. The percentage contributions of the interaction effects of ABC (pouring temperatures × titanium content × heat treatment method), AB (pouring temperature × titanium content) are respectively about 14%, 5%.

Table 3. Length of Crack Measurements on Nickel- Titanium Alloy Experiment

A	B	C	D	Treatment Combination	I	II	Total
-	-	-	-	(1)	7.037	6.376	13.413
+	-	-	-	a	14.707	15.219	29.926
-	+	-	-	b	11.635	12.089	23.724
+	+	-	-	ab	17.273	17.815	35.088
-	-	+	-	c	10.403	10.151	20.554
+	-	+	-	ac	4.368	4.098	8.466
-	+	+	-	bc	9.360	9.253	18.613
+	+	+	-	abc	13.440	12.923	26.363
-	-	-	+	d	8.561	8.951	17.512
+	-	-	+	ad	16.867	17.052	33.919
-	+	-	+	bd	13.876	13.658	27.534
+	+	-	+	abd	19.824	19.639	39.463
-	-	+	+	cd	11.846	12.337	24.183
+	-	+	+	acd	6.125	5.904	12.029
-	+	+	+	bcd	11.190	10.935	22.125
+	+	+	+	abcd	15.653	15.053	30.706

Source: 6.10 Problems; problem 6.15, chapter 6 of D.C. Montgomery. "Two – Level Factorial Designs". Design and Analysis of Experiments, International Student Version, 3rdedition (2013), p 294. www.wiley.com/go/global

In another development, the effect of titanium content (B = 3.9759) is the largest positive effect. This shows that increasing factor B (titanium content) from the low level (-) to the high (+) level will escalate the size of crack on the mechanisms for jet turbine engines and this will constitute a grave difficulty in the last fragment, because it may cause a non-recuperate disaster. The pouring temperature × heat treatment effect (AC = -4.0078) is the largest negative effect. This foregoing assertion implies that an increase in the quantity of interaction between pouring temperature (A) and heat treatment method (C) added to the process will deescalate the length of crack on the mechanisms for jet turbine engines and this will not constitute a stern difficulty in the last part, because it will lead to a recoverable problem in the event of any. The effect of the interaction between the four main effects pouring temperature × titanium content × heat treatment method × amount of grain refiner (ABCD = 0.0141) appears to be small relative to the main effects and the interaction effects of AB, AC and ABCD. In experiments concerning 2^4 factorial designs, it is pertinent to study the direction and magnitude of the factor effects to decide which variables are expected to be significant. The statistical technique which is generally employed to ratify this clarification is the analysis of variance technique. The ANOVA technique for this study is presented in Table V. The critical value of the test is obtained at $\alpha = 0.05$ level of significance. This value is given as $F_{0.05,\,1,\,16} = 4.49$. From Table V, one would observe in column five of the table that all the computed F_0 values for the main effects pouring temperature (A), titanium content (B), heat treatment method (C) and amount of grain refiner (D) are all greater than the critical value $F_{0.05,\,1,\,16} = 4.49$. This assertion implies that the null hypothesis is rejected in favor of the alternative hypothesis. This means that the factors in the experiment which includes pouring temperature (A), titanium content (B), heat treatment method (C) and amount of grain refiner (D) have significant effect on the length of crack on components of jet turbine engines. In a related development, one would observe in column five of Table V that the interaction between the factors effect of pouring temperature × titanium content (AB), pouring temperature × heat treatment method (AC), and pouring temperature × titanium content × heat treatment method (ABC), have significant effect on the length of crack on components of jet turbine engines.

Table 4. Factor Effect Estimates and Sum of Squares for the 2^4 Design of the Data in Table 3

Model term	Effect estimate	Sum of squares
A	3.0189	72.9089
B	3.9759	126.4607
C	-3.5963	103.4641
D	1.9578	30.6623
AB	3.4169	29.927
AC	-4.0078	128.4965
AD	0.08275	0.0548
BC	0.096	0.0737
BD	0.0473	0.0179
CD	-0.0769	0.047
ABC	3.1375	78.7513
ABD	0.098	0.0768
ACD	0.0191	0.0029
BCD	0.0356	0.0102
ABCD	0.0141	0.0016
Model term	Percentage	contribution
A		12.76962468
B		22.14895131
C		18.12121326
D		5.370346596
AB		5.241562524
AC		22.50551137
AD		0.009597943
BC		0.012908182
BD		0.003135094
CD		0.008231812
ABC		13.79289146
ABD		0.013451131
ACD		0.00050792

5. Analysis on the Regression Model for Predicting the Factor Effects in a 2^4 Factorial Design

In this subsection, we propose a regression model that would be used to predict crack length as a function of the significant main effects of pouring temperature (A), titanium content (B), heat treatment method (C), and amount of grain refiner (D), as well as the interaction of pouring temperature × titanium content (AB), pouring temperature × heat treatment method (AC) and pouring temperature × titanium content × heat treatment method (ABC), identified in section III. The regression model considered in this study is a multiple linear regression model with four predictor variables where the predictor variables represent the four factors considered in this study, that is, pouring temperature (A), titanium content(B), heat treatment method (C), and amount of grain refiner (D). The proposed multiple linear regression model consist of the interacting components identified in section III, that is, pouring temperature × titanium content (AB), pouring temperature × heat treatment method(AC), and pouring temperature × titanium content × heat treatment method (ABC). Thus, the regression equation is given by equation (10).

$$Y_i = \beta_0 + \beta_1 X_{11} + \beta_2 X_{22} + \beta_3 X_{33} + \beta_3 X_{33} + \beta_5 X_{11} X_{22} + \beta_6 X_{11} X_{33} + \beta_6 X_{11} X_{22} X_{33} + \varepsilon_i \tag{10}$$

where $i = 0, 1, 2, 3, 4$

Our analysis in Table IV indicates that the significant effects are A = 3.0189, B = 3.9759, C = -3.5363, D = 1.9578, AB = 3.4169 AC = -4.0078 and ABC = 3.1375. These values are employed to estimate the coefficients of the regression model (10) as already described in section III.

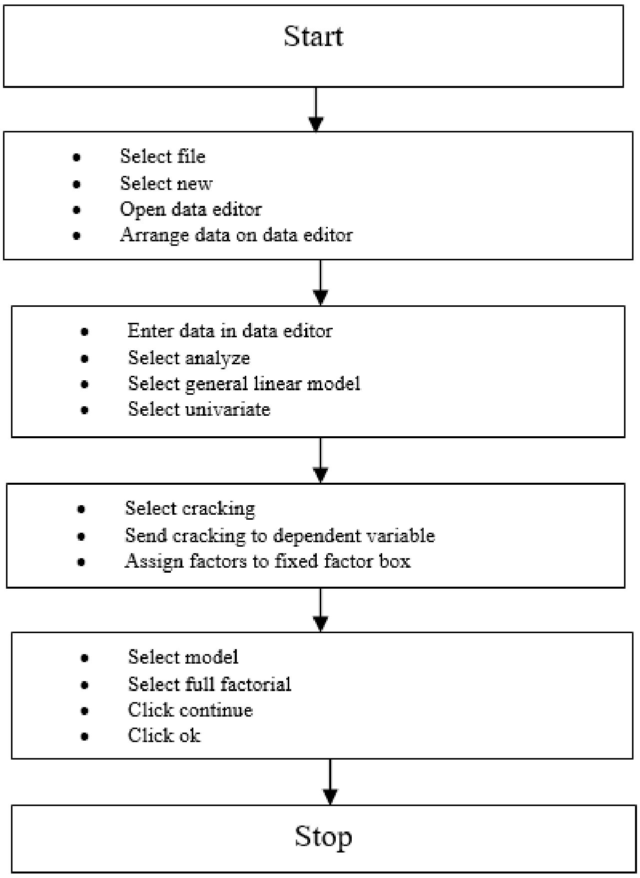

Figure 2. Chart showing steps in the analysis of the 2^4 factorial design using SPSS

The intercept β_0 in equation (10) is the average response and it is obtained from the overall mean of the data in Table III. Consequently, the estimated crack length as a function of the significant main effects of pouring temperature (A), titanium content (B), heat treatment method (C) and amount of grain refiner (A) as well as the interaction of pouring temperature × titanium content (AB), pouring temperature × heat treatment method (AC) and pouring temperature × titanium content × heat treatment method (ABC) is given by the equation below.

$$y = 11.9881+(3.0189/2)X_{11}+(3.9759/2)X_{22}-3.5963/2)X_{33} +(1.9578/2)X_{44}+(3.5169/2)X_{11}X_{22}-(4.0078/2)X_{11}X_{33}+(3.1375/2)X_{11}X_{22}X_{33}$$

The coded variables x_{11}, x_{22}, x_{33}, takes on values between -1 and +1. The predicted crack length at run (1) is

$$y=11.9881+(3.0189/2)(-1)+(3.9759/2)(-1)-(3.5963/2)(-1)+(1.9578/2)(-1)+(3.5169/2)(-1)-(4.0078/2)(-1)(-1)+ (3.1375/2) (-1) (-1) (-1)$$

$= 7.44575$

Since the observed value in two replicates is 13.413, the residual is e =13.413-7.44575 = 5.96725. The values of y and e for all sixteen observations are given in Table 6. Fig. 3 shows the normal probability plot for residuals. The points on the residual plot lies equitably handy to the straight line. This assertion offers credence to our conclusion in section IV that the main factor effects which include pouring temperature (A), titanium content (B), heat treatment method (C) and amount of grain refiner (D) as well as the interaction of pouring temperature × titanium content (AB), pouring temperature × heat treatment method (AC) and pouring temperature × titanium content × heat treatment method (ABC) are the only significant effects and that the causal statements of the investigation are fulfilled. Furthermore, we would observed that the 2^4 factorial design employed in this study on data for nickel – titanium used to make parts for jet turbine aircraft engines, in order to determine the effect of four factors on cracking which is a potentially serious problem in the final part, is very efficient and readily providing information about interesting effects, and how the multiple linear regression equation (10) strengthened the validity of the findings. In this study, we also noticed it is a multifactor experiment, since we employed four factors in the investigation, implying that the process permits the investigation of a number of factors with the undistinguishable accuracy as if the whole investigation had been dedicated to the analysis of only one factor. Also, one factorial experiment offers facts on interaction effects while the conventional one factor-at-a-time methodology needs a succession of experiments for achieving the latter. In another development, we wish to determine whether any factors affect variability in cracking. The foregoing is achieved by comparing all pairs of means for the main effects in the data and this is because the null hypothesis of the main effects was rejected. In this article, we employed two methods to compare pairs of means for the main effects to ascertain whether any of the main factors effect affects variability in cracking. These two methods are (i) Duncan's multiple range (DMR) test (ii) the least significance difference (LSD) method. The Duncan's multiple range (DMR) test is a widely used technique for comparing all pairs of means and was developed by [7]. To apply Duncan's multiple range (DMR) test, the factor averages are arranged in ascending order, and the standard error of each average is determine by equation (12) below.

$$S_{yi} = \sqrt{MSE/n} \tag{12}$$

From the ANOVA table of Table V, the MSE = 1.2904. Recalled, that the experiment is in two replicates of a 2^4factorial design, therefore n = 4 in equation (12). The error degree of freedom is N = 16, since the 2^4 factorial design has 16 treatment combinations. Ranking the factor averages in ascending order, we have

$$y_d = 8.756$$
$$y_c=10.277$$
$$y_b=11.862$$
$$y_a=14.963$$

The standard error of each factor average is $S_{yi} = \sqrt{1.294/4}$. From the table of significant ranges which can be found in any statistics textbook for 16 degrees of freedom and $\alpha = 0.05$, we obtain $r_{0.05}$ (2, 16) =3.00, $r_{0.05}$ = (3, 16) =3.15 and $r_{0.05}$ = (4, 16) =3.23. Thus, the least significant ranges are as follows.

$$a \text{ vs } d = 14.963 – 8.756 = 6.207 > 1.835\ (R_4)$$
$$a \text{ vs } c = 14.963 – 10.277 = 4.686 > 1.789\ (R_3)$$
$$a \text{ vs } b = 14.963 – 11.862 = 3.101 > 1.704\ (R_2)$$
$$b \text{ vs } d = 11.862 – 8.756 = 3.106 > 1.709\ (R_3)$$
$$b \text{ vs } c = 11.862 – 10.277 = 1.585 < 1.704\ (R_2)$$
$$c \text{ vs } d = 10.277 – 8.756 = 1.521 < 1.704\ (R_2)$$

Table 5. ANOVA for the Length of Crack on the Components for Jet Turbine Engines Experiment in a 2^4 Factorial Design

Source of variation	Sum of squares	DF	Mean square	F_0
A	72.9089	1	72.9089	904.577
B	126.4607	1	126.4607	1569.033
C	103.4641	1	103.4641	1283.674
D	30.6623	1	30.6623	380.426
AB	29.927	1	29.927	371.303
AC	128.4965	1	128.4965	1594.249
AD	0.0548	1	0.0548	0.6799
BC	0.0737	1	0.0737	0.9140
BD	0.0179	1	0.0179	0.222
CD	0.047	1	0.047	0.583
ABC	78.7513	1	78.7513	977.086
ABD	0.0768	1	0.0768	0.953
ACD	0.0029	1	0.0029	0.036
BCD	0.0102	1	0.0102	0.127
ABCD	0.0016	1	0.0016	0.019
Error	1.2904	16	0.08065	
Total	572.2461	31		

From the analysis, one would observed that there are significant variations between all pairs of treatment combinations of means of the main factor effects except *b vs c* and *c vs d*. This means that factors A and B, A and C as well as A and D affects variability in cracking, whereas B and C with C and D showed no indication of affecting variability in cracking.

The second method employed in this study to determine whether any factor affects variability in cracking is the least significance difference (LSD) method. In the LSD procedure, we simply compare the observed difference between each pair of averages to the corresponding LSD. This means that if $|y_{i.} - y_{j.}| > \text{LSD}$, we infer that the population means μ_i and μ_j vary. The LSD employed in this study is given by equation (13) below.

$$\text{LSD} = t_{\alpha/2,\ N-\alpha}\sqrt{2MSE/n} \tag{13}$$

We illustrate the procedure using main factor effects which include pouring temperature (A), titanium content (B), heat treatment method (C) and amount of grain refiner (D) and has been demonstrated already to be significant in the analysis of Table V. Hence, the LSD is computed as follows.

$\text{LSD} = t_{0.05,16}\sqrt{2(1.2904)/4} = 2.12\ (0.8963) = 1.90$

The five main effect factor averages are: $y_a = 14.963$, $y_b = 11.862$, $y_c = 10.277$, $y_d = 8.756$ and the differences in averages are

$$y_{a.} - y_{b.} = 14.963 - 11.862 = 3.101$$
$$y_{a.} - y_{c.} = 14.963 - 10.277 = 4.686$$
$$y_{a.} - y_{d.} = 14.963 - 8.756 = 6.207$$
$$y_{b.} - y_{c.} = 11.862 - 10.277 = 1.585$$
$$y_{b.} - y_{d.} = 11.862 - 8.576 = 3.106$$
$$y_{a.} - y_{b.} = 14.963 - 11.862 = 3.101$$
$$y_{c.} - y_{d.} = 10.277 - 8.576 = 1.521$$

From the differences in means as illustrated above, the differences are all greater than the LSD = 1.9, except for $y_{b.} - y_{c.} = 1.585$ and $y_{c.} - y_{d.} = 1.521$. This assertion offers credence to our conclusion on Duncan's multiple range (DMR) test that there are significant variations between all pairs of treatment combinations of means of the main factor effects except *b vs c* and *c vs d*. However, it is not always the case that Duncan's multiple range (DMR) test and the least significance difference (LSD) method must produce identical inference, but in most cases the two methods always gives the same conclusion.

Table 5. ANOVA for the Length of Crack on the Components for Jet Turbine Engines Experiment in a 2^4 Factorial Design

Treatment combination	Observed Value (y)	Estimated Value	Residual Value e
(1)	13.413	7.44575	5.96725
a	29.926	14.19305	15.73295
b	23.724	11.14225	12.58175
ab	35.088	18.44835	16.63965
c	20.554	10.99475	9.55925
ac	8.466	3.45145	5.01455
bc	18.613	8.41625	10.19675
abc	26.363	13.98175	12.38125
d	17.512	9.40355	8.10845
ad	33.919	16.15085	17.76815
bd	27.534	13.10005	14.43395
abd	39.463	20.40615	19.05685
cd	24.183	12.95255	11.23045
acd	12.029	5.40925	6.61975
bcd	22.125	10.37405	11.75095
abcd	30.706	15.93955	-14.93955

6. Conclusions

In this article, we employed the 2^4 Factorial Design to analyse data on nickel-titanium used to create components for a jet turbine engine. The aim here is to decide the consequence of four factors : pouring temperature (A), titanium content (B), heat treatment technique (C) and quantity of particle refiner (D) as well as the interaction of pouring temperature × titanium content (AB), pouring temperature × heat treatment method (AC), titanium content × heat treatment (BC), heat treatment × amount of grain refiner (CD) and pouring temperature × titanium content × heat treatment method (ABC) and pouring temperature × titanium content × heat treatment method × amount of grain refiner (ABCD) on the length of crack in the components. This is because cracking constitute a grave problem in the last fragment for the reason that it may cause an irrecoverable failure. Having employed the 2^4 Factorial Design, we observed that the four factors have a significant effect on cracking. This assertion was confirmed by a multiple linear regression model. We compared all pairs of means for the main factors effects in the data in order to determine which of the main factor effect affects variability in cracking and we achieved this feat with the deployment of the Duncan's multiple range (DMR) test and the least significance difference (LSD) method. These two multiple comparison methods produce the same result.

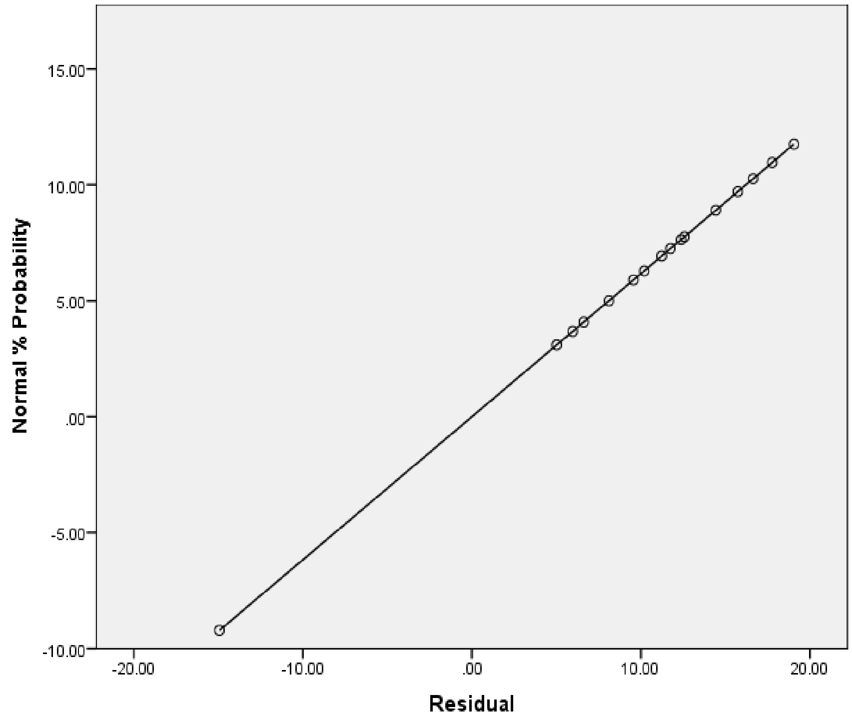

Figure 3. Normal Probability Plot for Residuals

We hope that the 2^4 Factorial Design employed in this study would draw the attention of captains of industries, organizations, the agricultural sector, the education sector to adopt and deploy the model in their respective sectors. Researchers may wish to explore the applications of other factorial experiments invoving more than four factors both at two stages as well as factorial experiments involving more than four factors each at three levels.

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