Intelligent Vibration Prediction of Thermally Stressed Rotating Functionally Graded Beams using Neural Networks Approach
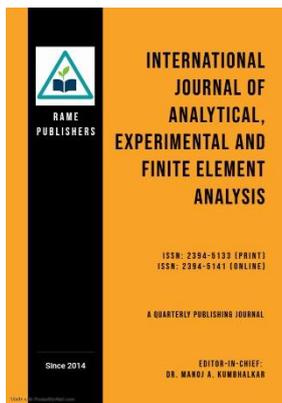
Uttam Kumar Kar[*], **Milan Banerjee**, **Pushpita Dasgupta**, **Vinay Kumar**, **P. S. Rao**

Department of Mechanical Engineering, Christian College of Engineering and Technology, Bhilai, India.

*Correspondence: ukkar093@gmail.com

Abstract: The rotational components are widely used in many engineering applications such as turbine blades, high compressor blades, gyroscopes etc. The vibration analysis of such components is very essential to understand their dynamic behaviour at complex environment. Present work mainly focuses on the development of a novel AI-based approach for the vibration analysis of a functionally graded (FG) rotating beam exposed to high temperature environment. To account the non-linearity and complex interaction between material compositions, centrifugal force and high thermal load feed forward multilayer perceptron based artificial neural network model is proposed. The vibration responses of FG beam for different grading indices, temperature changes and rotational speeds are first calculated using finite element based simulation and furthermore FE data is imported in ANN model for training and testing. After input-output interactions the vibration behaviour is predicted and compared with FE simulation with unknown inputs. The results show that the proposed intelligent approach is powerful tool to predict the dynamic response of thermally loaded rotating FG components especially at condition monitoring.

1. Introduction

Nowadays, the rotational components at high elevated temperature such as automobile shafts, compressor blades, centrigul pumps are widely used in many industry[1–4]. The dynamic behaviour of such components is very crucial for designer to understand the performance and operation life at harsh working conditions. To predict the vibration response with high accuracy and minimum computational time, an effective technique is necessary to develop.

Non-uniform temperature rise affect the dynamic performance of the rotating components. non-uniform temperature changes induces the non-linear thermal stresses and disturb the material compositions, leads to increase thermal instability[5–7]. Parida and Mohanty [8] examined the vibrational behaviour of rotating FG plate under non-uniform temperature changes. Chen et al. [9] reported comparable findings while examining rotating pre-twisted functionally graded (FG) sandwich blades operating in thermal environments. Furthermore, Zhang and Li [10] analyzed the nonlinear forced vibration behavior of pre-deformed rotating blades subjected to uniformly distributed gas pressure under a thermal gradient.

Various numerical and analytical approaches have been developed and reported in the literature to address the static and vibration response of thermally loaded rotating FG components. Shahba et al. [11] employed the finite element method to investigate the free vibration characteristics of spinning conical beams composed of functionally graded material. Entezar et al.[12] studied the stress distribution of rotor and disk with various thickness using refined finite element method. Heydarpour et al.[13] investigated elastic response of FG cylindrical shells in thermal environment using differential quadrature method.

Furthermore, due to the highly coupled and non-linear systems that arise from the interaction of rotational and thermal processes, analytical and traditional numerical methods are computationally very expensive and often have limited accuracy and generalization capabilities [14–16]. For complex thermal-mechanical coupling problems, analytical solutions are often limited to idealized geometries and boundary conditions, while traditional finite element methods (FEM) can require significant computational resources, especially when a large range of operational parameters is being investigated [17–19]. To overcome the limitations of the traditional technique, it is necessary to develop an advance AI based approach to capture the vibration behaviour of the componnets in such complex conditions. Recently, artificial intelligent based neural network and fuzzy logic techniques are emerged as effective tool for capturing dynamic response of nonlinear complex systems [20]. The advantages of these technique is that , it can capture dynamic behaviour of components with high accuracy and minimum computational efforts by learning simulation data sets. The data set contain input and output of the system. Neural network perform training and testing of these data set using effective activation function and easily predict the dynamic characteristics of the nonlinear systems. Therefore, to understand the effect of temperature, centrifugal force, material compositions and other non-uniform conditions on the vibration characteristics of the thermally stressed rotating FG beam , AI tool is best alternative for traditional numerical technique. Recently, Siavashi et al. [21] studied the vibration and stability behaviour of the nano shell structure using intelligent model with diferent shape and boundary conditions. Yang et al. [22] used intelligence approach to identify the cracks in railway tunnel during earthquakes. High prediction accuracy was achieved while comparing actual cracks in tunnel structures.

This research aims to develop a novel intelligent-based approach for the extensive vibration response of thermally-stressed rotating FG beams. The main goal is to develop a predictive model that can accurately identify the vibrational properties of the beam, including the complex interactions between rotational and thermal effects. The objective is to present an effective neural network model that can accurately and computationally represent the non-linear interactions governing the system dynamics, compared to traditional methods. The developed methodology can serve an effective tool for engineers and researchers involved in the design, optimization, and condition monitoring of rotating machinery operating in high-temperature environments. The remaining sections of this paper explain the mathematical model, the NN model, results and discussion, followed by the conclusions and future scopes of the present work.

2. Mathematical Model

The schematic of functionally graded rotating beam with length L, thickness h is shown in Fig. 1. The beam is exposed to uniform thermal environment.
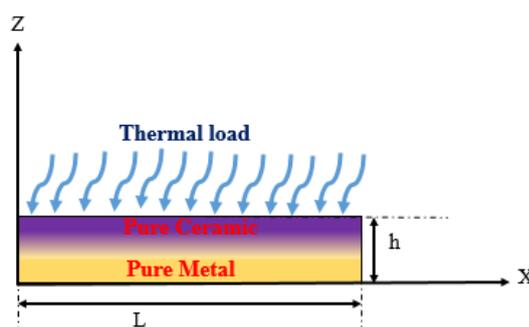

Figure 1. Schematic of thermally stressed FG beam

2.1 Material Model

The functionally graded beam is graded in thickness direction as SiC as ceramic and Aluminium as a metal. The upper phase contains pure ceramic and the lower phase contains pure metal. The effective material properties like elastic modulus, density and thermal expansion coefficient are obtained using the power law, which is expressed as:

$$P(z) = P_c(z) + (P_m(z) - P_c(z))\left(0.5 + \frac{z}{h}\right)^{n_z} \tag{1}$$

Where $P(z)$, $P_c(z)$ and $P_m(z)$ represent the effective material property, ceramic material and metal, respectively. The grading index in thickness direction is indicated as n_z.

2.2 Dynamic Model

Let transverse displacement w(x,t) and cross-section rotation φ(x,t) be the primary fields in the beam. The beam is also subjected to thermal load and rotational effect, therefore total axial stress is $N_{Total} = N_{th} + N_R$.

Where

$$N_{th} = E_{eff}\, \alpha_{eff} A \Delta T \tag{2a}$$

$$N_R = \int_x^L \rho_{eff} A\, \Omega^2(s)ds = \frac{1}{2}\rho_{eff}A\, \Omega^2(L^2 - x^2) \tag{2b}$$

The governing equation of motion including thermal and rotational effect using Timoshenko beam theory can be obtained as:

$$\rho_{eff}A\frac{\partial^2 w}{\partial t^2} - \frac{\partial}{\partial x}\left[k_s G_{eff} A\left(\varphi - \frac{\partial w}{\partial x}\right)\right] - \frac{\partial}{\partial x}\left(N_{Total}\frac{\partial w}{\partial x}\right) = q(\text{x}, \text{t}) \tag{3a}$$

$$I_r\frac{\partial^2 \varphi}{\partial t^2} - \frac{\partial}{\partial x}\left(E_{eff}\, I\frac{\partial \varphi}{\partial x}\right) + \left[k_s G_{eff} A\left(\varphi - \frac{\partial w}{\partial x}\right)\right] = m(\text{x}, \text{t}) \tag{3b}$$

Where $q(\text{x}, \text{t})$ is transverse distributed load, I_r indicates rotary inertia per unit length and k_s is shear correction factor. The governing equations are converted to their weak form and discretized using standard Timoshenko beam finite elements. Assembly yields the semi-discrete system without external load as:

$$M\ddot{d} + (K + K_G)d = 0 \tag{4}$$

In which M, K and K$_G$ are the mass and stiffness matrices. The global DOF vector is represented as d. The above equation is solved to obtain the eigen value and eigen vector.

2.3 Neural Network Model

A feed-forward neural network maps an input vector x to an output y through a series of weighted transformations. For a single hidden layer network, the mathematical model can be expressed as:

$$y_h = f_1(W_1 x + b_1) \tag{5}$$

$$y_o = f_2(W_2 x + b_2) \tag{6}$$

Where

W_1, W_2 are weight matrices, b_1, b_2 are bias vectors, f_1 is the hidden layer activation function and f_2 is the output activation function. The training objective is to minimize the prediction error:

$$min_{W,b}\|y_{pred} - y_{target}\|^2 \tag{7}$$

3. Results and Discussions

In present study, the cantilever rotating beam with length L = 300mm, width b = 60mm and thickness h = 15mm is considered , which are standard dimensions of a high speed compressor blade[23]. A user interactive programme is developed and implemented in MATLAB (v2018a). The temperature and speed range is decided based on practical application. The material properties are taken from literature [14,24].

3.1 Validation Study

To validate the present approach, a FG rotating cantilever beam is considered with length L=1000mm, width b= 20mm and thickness t = 20mm. The alumina (ceramic) and steel (metal) are considered as FG material. The material properties are taken as same as reference [25]. Table 1 shows the comparison of fundamental frequency of rotating FG beam for various grading index value.

Table 1. Comparison of natural frequency with different grading index

Grading index (n$_z$)	Present (FE) results	Ref. [25] results	Error(%)
0	34.965	35.677	2.03
4	18.987	19.464	2.51
8	17.954	18.309	1.97

From Table 1, it is clear that the present approach is in very good agreement with published results. In the present study, finite element method has been employed and Rayleigh-Ritz method has been employed by the published paper. The published data has been found to have an error of less than 3%.

3.2 FE simulation results

Fig. 2 illustrated the variation of natural frequency of rotating beam with different temperature change and FG material compositions. Due to the temperature-dependent degradation of the elastic modulus, both SiC and Al become less rigid as the temperature increases from 50 °C to 300 °C, and the resulting thermal expansion generates compressive thermal stress. These combined effects significantly reduce the effective bending rigidity of the beam, leading to a decrease in the natural frequency.
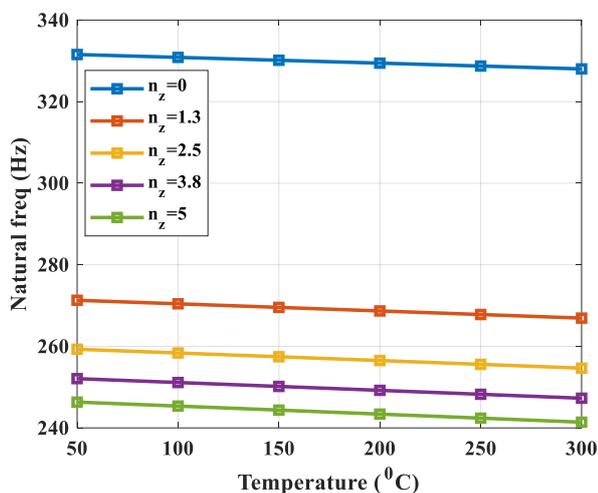

Figure 2. Variation of natural frequency of FG rotating beam for different temperature change and gradation index

The findings also reveal that the FG material composition affects the beam's frequency values. Lower power-law indices (ceramic-rich) maintain higher natural frequencies and exhibit better thermal stability because SiC is more effective than aluminum in maintaining stiffness at higher temperatures. The fact that FGMs behave differently at moderate and high temperatures is evidenced by the non-linear decrease in stiffness at higher thermal levels. Overall, the results show that the ceramic-rich gradient helps maintain structural integrity and vibration resistance under thermal loading, and that temperature is a key factor in the dynamic behavior of rotating FGM beams.

Similarly, the effect of rotational speed of high compressor blade on vibration response of FG beam is shown in Fig. 3. As the rotational speed increases (from 500 rad/s to 2000rad/s), the natural frequency tends to increase due to the additional centrifugal stiffening effect, which introduces additional axial tension along the beam length.
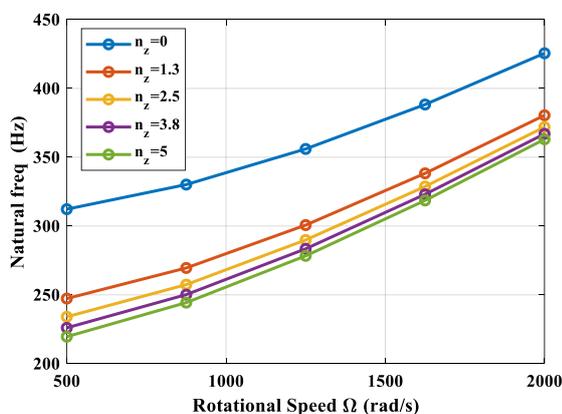

Figure 3. Variation of natural frequency of FG rotating beam for different rotational speed and gradation index

This tensile force generated by rotation effectively increases the overall stiffness of the beam, shifting the natural frequency to higher values, especially in the higher speed range. However, the extent of this increase depends significantly

14

on the FG power-law index. The FG beam with higher SiC composition exhibits better natural frequencies across all rotational speeds, because ceramic material offers more stiffness and better resistance to rotational stiffening effect. On the other hand, rotating components with a higher metal content (higher n_z) are less rigid and therefore have lower natural frequencies. Overall, the trend shows that by choosing the appropriate gradation value of FGMs, designers can tune the dynamic performance of the rotary instruments.

3.3 Neural Network Predictions

To train and validate the neural network model, FE simulation data were used to predict the vibration response of thermally stressed rotating functionally graded beams. Total 27 datasets were prepared , in which 70% data were used for training, 15% data were for testing and rest 15% data were used for validation. The multi-layer perceptron back propagation algorithm with 10 hidden neurons were used. The architecture of neural network 3-10-1 is illustrated in Fig. 4.
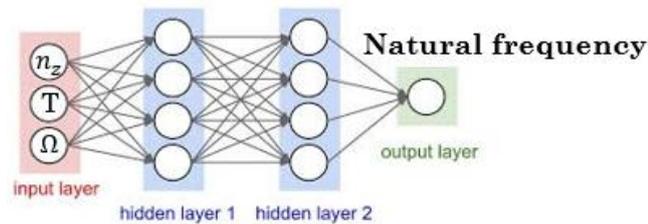

Figure 4. Architecture of neural network

The error plot, regression plot, and comparison with FE results are illustrated to prove the accuracy of proposed intelligent model. The difference between the neural network predictions and the corresponding FE findings across all test samples is depicted in the error plot in Fig. 5. This plot illustrates the learning efficiency of neural network by showing the size and distribution of prediction errors. When the error band is small and exhibits only minor variations, that means the neural network effectively captures the non-linear relationships between rotational speed, temperature gradient, material gradation, and vibration response
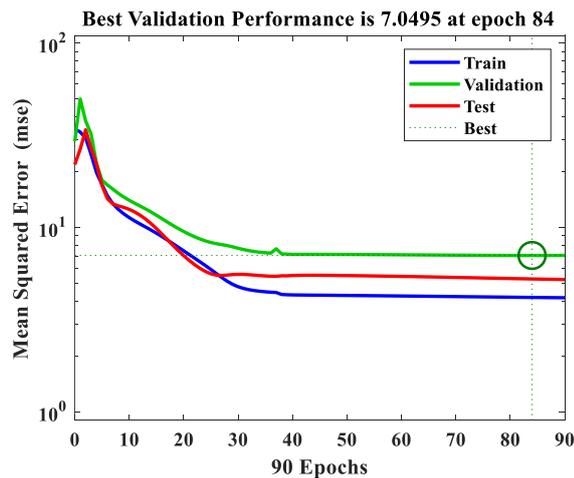

Figure 5. MSE error plot

To further analyze the performance of the neural network, regression plots of the training, validation, and testing datasets were used, as shown in Fig. 6. These plots show the network output and the corresponding FE value for each learning step. Correlation coefficients close to 1 ($R \approx 1$) indicate good performance. Moreover, to demonstrates the effective learning capacity, minimum overfitting and generalization with higher accuracy and stability of the proposed network for all data set is determined by different regression plots. The smooth convergence of regression curve for training , testing and validation indicates the accuracy and stability of the proposed neural network model.
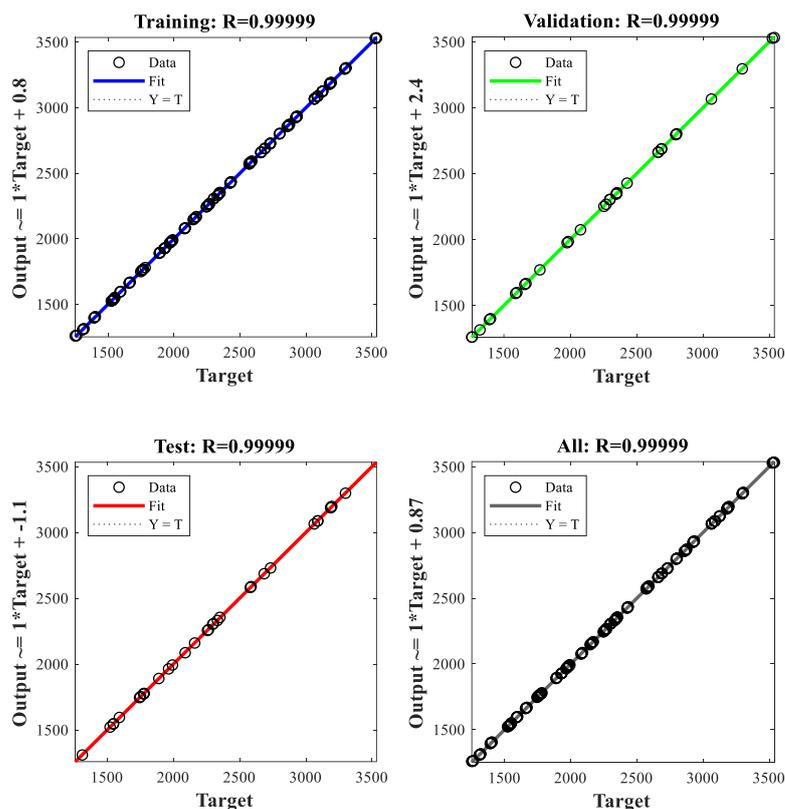

Figure 6. Regression plot of proposed NN model

After training and testing of the input-output data set, the unknown input value of grading index, temperature and rotational speeds are varied and predict the natural frequency using proposed neural network model. The predicted values are compared with original FE simulation results as shown in Table 2. It is evident that predicted results are very close to FE results. Approximately less than 5% error is found between these AI and traditional approach. This results indicates the accuracy and stability of proposed intelligent technique to capture the dynamic behaviour of the thermally stressed rotating FG components. Therefore, it can be concluded that AI based prediction technique is best approach for predicting vibrational characteristics of the complex structures with reduced computational load and time.

Table 2. Comparative results of proposed NN model and FE simulation

Input parameters			Output parameters		Error
n_z	T (°C)	Ω (rad/s)	NN Output	FE output	(%)
1.5	25	800	296.79	283.12	4.60
2.5	35	1500	400.16	391.95	2.05
0.5	105	1400	431.42	410.97	4.74
0.25	120	750	351.00	335.31	4.47
0.3	235	560	327.01	318.15	2.70
4	220	1750	438.44	425.38	2.97
4.6	230	1440	373.17	368.25	1.31

4. Conclusions

Present work investigated the vibration characteristics of a rotating functionally graded beam subjected to high thermal and centrifugal load. Initially, the effect of rotational speed, power law index and temperature changes on natural frequency of the FG components were calculated using finite element approach. It was observed that increasing of grading parameter

reduces the fundamental frequency of the FG beam due to higher metal content. Similarly, higher rotational speed exhibits greater fundamental frequency. On other hand, higher temperature reduces the rigidity of the FG beam, consequently decrease the free vibration performance of the FG rotating beam.

Furthermore, to reduce the computational cost and rapid prediction of natural frequency, an effective AI based feed forward neural network model was developed by using back propagation algorithm. The FE simulation data was used to train the proposed network and different error plots were drawn to show the accuracy of model. It was observed that proposed NN model effectively predict the dynamic parameter with high efficiency and low computational time.

This work can be extended to multi-objective optimization of material combinations, experimental validation and exploration of advanced composite material like porous graphene material for rotating components.

References

[1] Pacheco-Chérrez J, Cárdenas D, Delgado-Gutiérrez A, Probst O. Operational modal analysis for damage detection in a rotating wind turbine blade in the presence of measurement noise. Compos Struct 2023;321. https://doi.org/10.1016/j.compstruct.2023.117298.

[2] Liu N, Mo H, Wang H, Huang A, Zhang J, Wang Z, et al. Simulation and experimental analysis of diesel engine shafting angular misalignment for online fault detection. Meas J Int Meas Confed 2025;252. https://doi.org/10.1016/j.measurement.2025.117406.

[3] Hussain S, Ghopa WAW, Singh SSK, Azman AH, Abdullah S. Experimental and Numerical Vibration Analysis of Octet-Truss-Lattice-Based Gas Turbine Blades. Metals (Basel) 2022;12. https://doi.org/10.3390/met12020340.

[4] Bhupesh Sonkar, Amit Sarda RB. Enhancing the Performance of Turbine Blades through CAD-Based Design Optimization and Finite Element Analysis: A Comprehensive Review. Int J Anal Exp Finite Elem Anal 2023;10:35–7.

[5] Shah S, Luintel MC, Bhattrai S. Comprehensive review on vibration response and dynamic analysis of rotating blades under thermal environment. Int J Dyn Control 2025;13. https://doi.org/10.1007/s40435-025-01776-6.

[6] Kumar Kar U, Srinivas J. Vibration analysis of Bi-directional FG-GNPs reinforced rotating micro-beam under Thermo-mechanical loading. Mater Today Proc 2022;78:752–9. https://doi.org/10.1016/j.matpr.2022.10.227.

[7] Waleed Khalid Mohammed, Khamees N. Abdulhaleem, Shwan H. Said QFH. Finite Element Analysis of Self-Compacted Concrete Filled Steel Tube Columns Exposed to High Temperatures. Int J Anal Exp Finite Elem Anal 2023;10:1–10.

[8] Parida S, Mohanty SC. Free vibration analysis of rotating functionally graded material plate under nonlinear thermal environment using higher order shear deformation theory. Proc Inst Mech Eng Part C J Mech Eng Sci 2019;233:2056–73. https://doi.org/10.1177/0954406218777535.

[9] Chen Y, Ye T, Jin G, Li S, Yang C. Vibration analysis of rotating pretwist FG sandwich blades operating in thermal environment. Int J Mech Sci 2021;205. https://doi.org/10.1016/j.ijmecsci.2021.106596.

[10] Zhang B, Li Y. Nonlinear vibration of rotating pre-deformed blade with thermal gradient. Nonlinear Dyn 2016;86:459–78. https://doi.org/10.1007/s11071-016-2900-4.

[11] Shahba A, Attarnejad R, Zarrinzadeh H. Free vibration analysis of centrifugally stiffened tapered functionally graded beams. Mech Adv Mater Struct 2013;20:331–8. https://doi.org/10.1080/15376494.2011.627634.

[12] Entezari A, Kouchakzadeh MA, Carrera E, Filippi M. A refined finite element method for stress analysis of rotors and rotating disks with variable thickness. Acta Mech 2017;228:575–94. https://doi.org/10.1007/s00707-016-1727-x.

[13] Heydarpour Y, Malekzadeh P, Golbahar Haghighi MR, Vaghefi M. Thermoelastic analysis of rotating laminated functionally graded cylindrical shells using layerwise differential quadrature method. Acta Mech 2012;223:81–93. https://doi.org/10.1007/s00707-011-0551-6.

[14] Kar UK, Srinivas J. Stability and vibration control of electrostatically excited functionally graded microresonator using nonlinear observer based sliding mode controller. Phys Scr 2024;99. https://doi.org/10.1088/1402-4896/ad79ac.

[15] Wang J, Chen W, Wang X, Liu N. Stability Analysis System of Functionally Graded Material Beams Based on Machine Learning. Procedia Comput Sci 2025;262:1002–10. https://doi.org/10.1016/j.procs.2025.05.135.

[16] Zheng F, Duan R, Chen X, Lu Y. Machine learning, numerical, and analytical approaches for vibration prediction of porous gradient piezoelectric beams under traveling force. Meas J Int Meas Confed 2025;245.

https://doi.org/10.1016/j.measurement.2024.116565.

[17] Nikita A. Duble and A. D. Diwate. Design and Finite Element Analysis of Differential Cover for Rear Drive axle of a Light Commercial Vehicle (LCV). Int J Anal Exp Finite Elem Anal 2020;7:53–60.

[18] Rushikesh Salunkhe, Shriram Sonawane, Shardul Bhalerao NS. Vibration Analysis for Bending of Vertical Plates in PVC Scrap Filtration Machine. Int J Anal Exp Finite Elem Anal 2019;6:79–86.

[19] Rishabh Tamrakar, Chandrashekhar Sahu PSR. Multi-Objective Optimization and Stress Analysis of an Automotive Upper Control Arm of Double Wishbone Suspension. Int J Anal Exp Finite Elem Anal 2025;12:1–5.

[20] Yildirim S. Free vibration of axially or transversely graded beams using finite-element and artificial intelligence. Alexandria Eng J 2022;61:2220–9. https://doi.org/10.1016/j.aej.2021.07.004.

[21] Siavashi M, Dardel M, Pashaei MH. Nonlinear stability and vibration analysis of fluid-conveying nanochannel scroll shells using an adaptive neuro-fuzzy inference system. Thin-Walled Struct 2026;218. https://doi.org/10.1016/j.tws.2025.113931.

[22] Yang L, Xin C, Wang Z, Yu X, Hajirasouliha I, Feng W. Seismic resilience analysis of high-speed railway tunnels across fault zones using ensemble learning approach. Undergr Sp 2025. https://doi.org/10.1016/j.undsp.2025.04.011.

[23] Shi H. A parametric blade design method for high-speed axial compressor. Aerospace 2021;8. https://doi.org/10.3390/aerospace8090271.

[24] Kar UK, Srinivas J. Frequency analysis and shock response studies in bidirectional functionally graded microbeam with thermal effects. J Brazilian Soc Mech Sci Eng 2022;44. https://doi.org/10.1007/s40430-022-03615-7.

[25] Ramesh, M. N. V. NMR. Free Vibration Analysis of Rotating Functionally-Graded Cantilever Beams. Int J Acoust Vib 2014;19:31–41.

RAME PUBLISHERS