Investigation of Stresses in Thin Rimmed Spur Gear Tooth using FEM

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***Abstract—* In this paper results are presented from a two-dimensional finite element stress analysis of three tooth sector of spur gear and the boundary conditions are provided at the radial end of the teeth of the rim. The different diametric ratio from 1.02 to 2 is used to get the stress variation. In the analysis of this three tooth sector the Maximum Principle stresses and von mises stresses at the root and fillet are in consideration. The main objective of this paper is to find out the minimum thickness of spur gear for different diametric ratio Md using analytical results by Lewis equation and FE results. In this study the CATIA software used for the geometrical construction and stress analysis is accomplished by commercial finite element package MSC PATRAN and NASTRAN.**

***Keywords—* Diametric ratio Md, Maximum Principle Stress analysis, Finite element method, Three tooth sector of spur gear.**

I. Introduction

Gears are indispensible in the transmission of mechanical power from machines to automotive and the aerospace engines. The gears are most commonly employed as power transmission elements. Among the gears, spurs gear most commonly used gears. Though the spur gears have been standardized for the profile requirements, the material reduction for the gear can be done if a rimed gear instead of a standard full gear is used. Especially industries like aerospace the mass reduction is the very important aspect when optimizing design, hence such application often used rimed gears as the maximum stresses in the gear always occurs at the root.

The gear body below the root has significant low stresses and is unlikely to fail. Hence removal of material from this portion can significantly reduce the weight of gear. However, the amount material should be reduced is the matter of investigation and hence it was decided to carry out the investigation on various stresses for the different thickness of rim.

I. Finite Element Analysis of Three Tooth Sector of Thin Rimmed Spur Gear

This investigation has been done using FEA for obvious advantages it offers. G.Mallesh et al [6] have discussed Bending stresses in thin rim spur gear tooth fillets and root areas differ from the stresses in solid gears due to rim deformations. Rim thickness is a significant design parameter for these gears. Hence the rim thickness have been established as diametric ratio which has defined as the ratio of Deddendum diameter to internal rim diameter of spur gear (diametric ratio Md = Dd/Dr).

A spur gear with following specification has been chosen as follows.

A. *Dimensions of spur gear*

Teeth number of spur gear = n= 50
Pressure angle = Φ =20°
Module = m = 4mm
Pitch circle diameter = D_P =200mm
Addendum height = ha= 4mm
Deddendum height = hd =5mm
Addendum circle diameter = Da = 208mm
Deddendum circle diameter=Dd=190mm
Circular pith= Pc=12.57mm
Tooth thickness=t=6.283mm
Fillet radius at root= 1.6mm
Face width=b=40mm
Power=P=7.5kw
Speed =N=750rpm
Young's module=E=210e3
Poisson's Ratio= 0.3
Pressure= Pi = 1 Mpa.

B. Model of three tooth sector of spur gear

In this work spur gear with three tooth sector s will be used for analysis. Where the internal rim diameter changes as per the diametric ratio Md = 1.02 to 2.00. The model is shown in fig.1, in which CATIA V17 is used for modelling the spur gear. The model is prepare in the three tooth sector because the Maximum Principle Stresses at the tooth fillet of whole gear and at the fillet of three tooth sector is approximately same. The three teeth sector of spur gear is shown in fig.2.

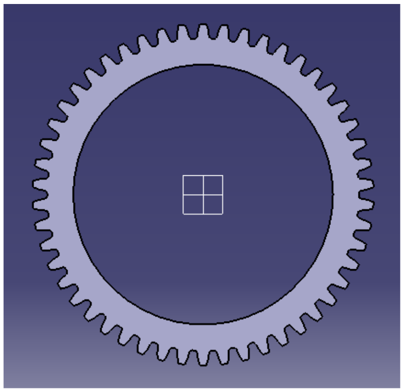

Fig.1Model of whole thin rimmed Spur Gear with 50 teeth

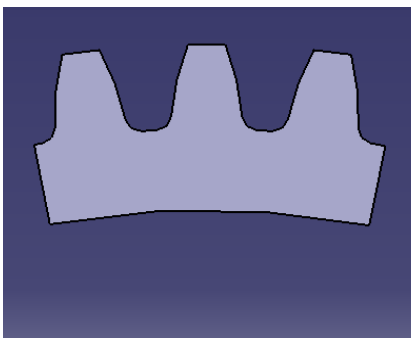

Fig.2Three teeth Sector of Spur gear

C. Meshing

Meshing is the integral part of computer - aided engineering (CAE) analysis process. The mesh influences the accuracy, convergence and speed of solution. Furthermore, the time it takes to get result from a CAE solution. Therefore, the MSC PATRAN software which having smart size option is to control size of element and nodes is used.

The meshed three tooth sector of spur gear model using element type 2D Shell. This element type is 2D stress having element count 191and nodes 159 and finer meshing is applied to model as shown in figure3.

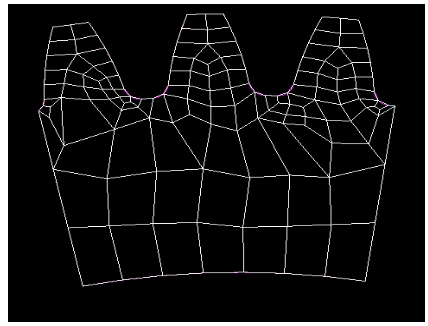

Fig.3 Meshed model of Sector of Spur gear

D. Loading

For assigning the load, the nodal force is applied by assigning the component in each direction which is the tangential and radial force respectively. Following are the theoretical stresses by using Lewis equation.

Ft = σ P_c y b

σ = 13.98 N/mm^2

y = Lewis form factor= 0.13576

T = Torque Transmitted =95.49 Nm

Ft = tangential tooth load =954.9 N

Fn = Normal tooth load =1016.18

E. Boundary conditions

For the case of three tooth sector of spur gear one internal radial end is fixed at the six D.O.F. hence the internal rim with all nodes on it was fixed i.e. $T_x = T_y = T_z = R_x = R_y = R_z = 0$.

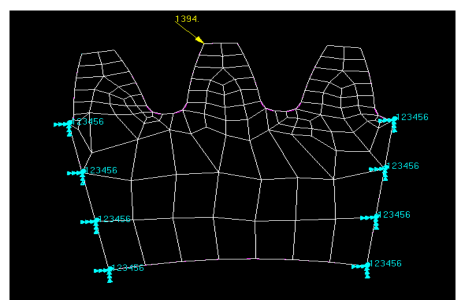

Fig.4 Gear teeth with Load and Boundary conditions

II. Results and Discussion

Diametric ratio Md was varied from 1.02 to 2.00. Table number 5.2 gives the dimension of internal rim diameter.

Table1: Internal diameter of Rim as per Diametric ratio Md

Diametric ratio Md	**Internal diameter of Rim**
1.02	186.27
1.06	179.25
1.10	172.72
1.14	166.67
1.18	161.01
1.22	155.73

1.26	150.79
1.30	146.15
1.34	141.79
1.38	137.68
1.42	133.80
1.46	130.13
1.50	126.67
1.54	123.37
1.58	120.25
1.62	117.28
1.66	114.45
1.70	111.76
1.74	109.19
1.78	106.74
1.82	104.39
1.86	102.15
1.90	100.00
1.94	97.93
1.98	95.95
2.00	95.00

As per the table number 1 each case was analysed using MSC NASTRAN and MSC PATRAN and the deflection and stress pattern was noted. It was found that the variation in the stresses in gear for all cases is nearly close to each other. A sample case discussed was for Md = 1.26.

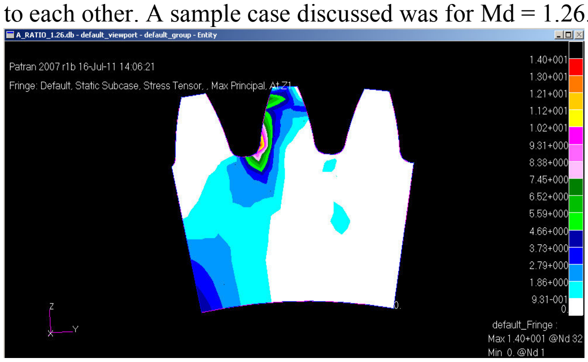

Fig 5 Stress pattern for Md=1.26

The figure shows the stress pattern. Maximum Principle Stress pattern obtained for the case of Mb=1.26. As seen the figure 5 the stresses at root of teeth is highest it is equal to 13.97 N/mm^2 which is approximate to the stress obtained as per the Lewis equation for gear design.

The stress drops along the radial direction and circumferential variations. The radial variations also shows some equistress contours concentrated with the root point. Decrease in the maximum Principle stress is faster along the circumferential directions than in the radial directions, as expected along the boundary of tooth at which the load is applied. The variation is similar to the cantilever case with the tip having minimum stress and tooth maximum. Due to the filleting there is much stress concentration at the root.

Table 2 Maximum Principle Stresses from tip to fillet of tooth

Radial distance to tooth tip (mm)	Maximum Principle Stress
1	13.97
3	10.40
4	7.15
5	4.48
6	7.23
7	3.36

The figure 6 shows the variation of stresses from the root to tip of tooth which clearly indicates the stress reduction.

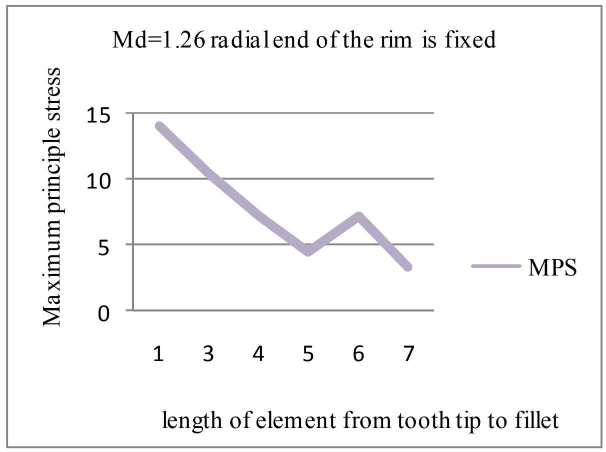

Fig.6 Maximum Principle Stresses from Tip to Fillet of tooth

The graph shown in figure 6.3 describes the variation of Maximum Principle Stress along the radial direction from tip to the radially inward element on the rim. The variation shows a quick decay in elements in selected region. The figure 6.4 shows the animated view of deformed view of tooth indicating a deflection is maximum at the tip.

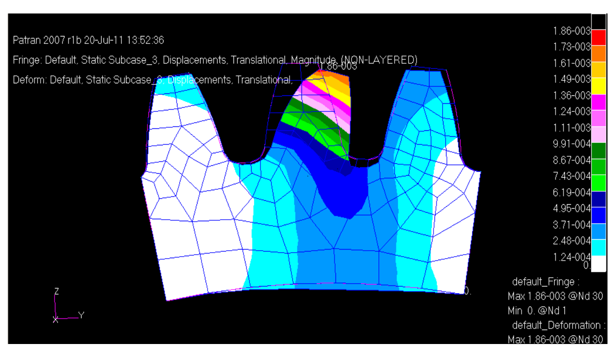

Fig.7 Deflection is Maximum at the Tooth tip

Figure 8 is a graph showing the deflection of nodes on the active face of gear tooth, i.e. the face on which load is applied which indicates a non linier absolute deflection. Results for deflection from fillet to tip of tooth are shown in table 3.

Table 3 Deflection from Fillet to Tooth tip

Location of node at length from fillet to tip of tooth	Node no.	Deflection
96.28	32	2.80×10^{-4}
97.87	36	4.71×10^{-4}
99.44	35	6.99×10^{-4}
100.98	34	9.66×10^{-4}
102.51	33	12.0×10^{-4}
103.99	30	17.0×10^{-4}

The graphical representation of resultant deflection of table 3 is illustrated as follows

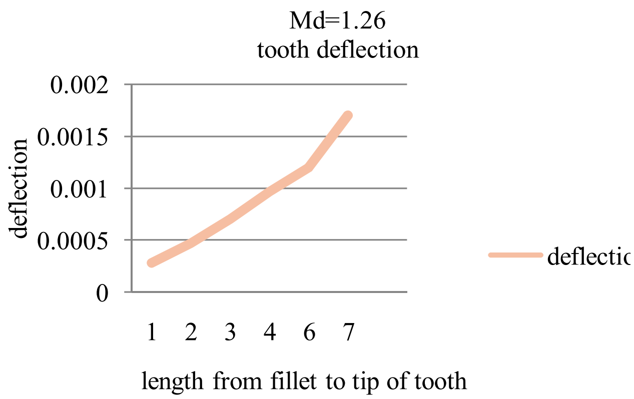

Fig.8 Deflection of tooth from fillet to Tooth tip

The report in table 3 for some of the elements of interest has been shown for the reference. This kind of analysis was done by varying the diametric ratio as mentioned. The following Table 6.6 gives the variation of maximum stresses and von mises stresses for varied diametric ratio as shown.

Table 4 Values of Maximum Principle Stress and Von mises Stress

Sr.No.	Md	Maximum Principal Stress	Von Mises Stresses
1	1.02	14.6	14.78
2	1.06	13.9	14.10
3	1.10	14.1	14.19
4	1.14	13.4	13.50
5	1.18	15.0	14.88
6	1.22	13.9	13.84
7	1.26	14.0	13.93
8	1.30	14.1	13.88
9	1.34	13.9	13.83
10	1.38	12.8	12.28
11	1.42	14.1	14.19
12	1.46	13.5	13.57
13	1.50	13.9	13.80
14	1.54	14.4	14.25
15	1.58	13.6	13.33
16	1.62	14.9	14.45
17	1.66	14.2	14.05
18	1.70	13.8	13.57
19	1.74	14.2	14.00
20	1.78	14.5	14.27
21	1.82	13.1	13.15
22	1.86	14.0	13.89
23	1.90	12.8	12.37
24	1.94	14.0	11.72
25	1.98	14.1	11.98
26	2.00	15.6	11.18

Following graph explains the nature of Maximum Principle Stresses obtained by changing diametric ratio as per table 4.

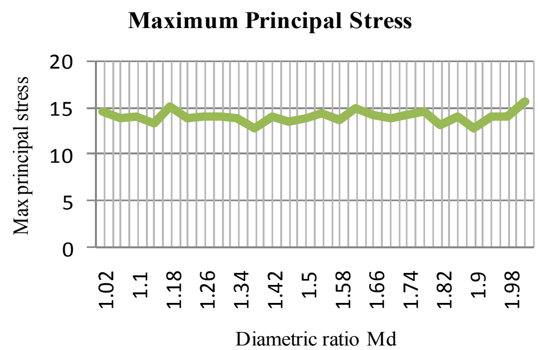

Fig.9 Maximum Principal Stresses with reference to Diametric ratio Md

Following graph explains the nature of von mises obtained by changing diametric ratio as per table 4.

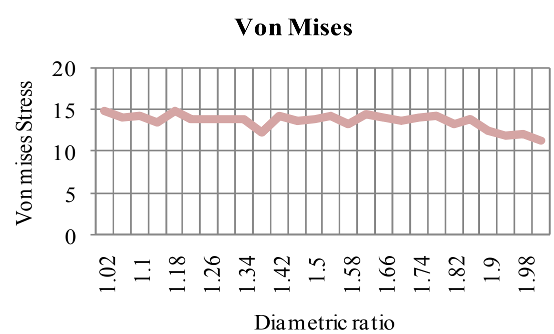

Fig.10 Von mises Stress with reference to Diametric ratio Md

Figure 9 and10 shows the variation of maximum principle stresses and von mises stress along with the variation of diametric ratio Md. It can be observed from the graph that the maximum principle stresses and von mises stresses remains almost constant through there is a trained of drop in maximum principle stress. This rate of decrease is small and insignificant and hence it can be established that the augmenting the rim thickness contributes to the reduction in the stresses and hence the strength of gear.

The variation of maximum principle stress along the diametric ratio for this case shows almost constant maximum principle stress for all cases. This is shown in figure9.

III. CONCLUSION

The spur gear for the different diametric ratio of Md, have been analysed by applying constraint in radial directions of every spur gear for 26 different rim thicknesses.

As discussed in the previous, the variation of the stress with the maximum principle stresses and the von mises stresses are similar for both the cases, so also the values of both these parameters are matching.

As observed from the graph, the variation of stresses (both Maximum principle stresses and the Von mises) beyond 1.2 is significant.

Hence increasing the rim thickness beyond the diameter ratio 1.2 does not contribute much to the strength of the gear. Hence the thin rim gear should have the minimum thickness as guided by the diametric ratio of 1.2 from the strength criteria.

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